

MTS 101

Elementary Mathematics

COMPLETE COURSE MATERIAL
Combined Past Questions, Solutions & Lecture Notes

Federal University of Technology, Akure

Topics: Set Theory | Real Numbers | Mathematical Induction | Sequences & Series | Trigonometric Functions | Circular Measure | Binomial Theorem | Quadratic Equations | Polynomials | Partial Fractions

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MTS 101 – COMBINED PAST QUESTIONS (CamScanner)

SECTION: COMBINED PAST QUESTIONS

This document contains combined past questions for MTS 101 Mathematics course.

Topics covered include: Set Theory, Real Numbers, Mathematical Induction, Sequences and Series, Trigonometric Functions, Circular Measure, Binomial Theorem and Expansion, Quadratic Equations and Polynomials, and Partial Fractions.

The questions are compiled from past examination papers to aid students in their revision and preparation for the MTS 101 examination at the Federal University of Technology, Akure.

Students are advised to attempt all questions and compare their answers with the provided solutions where available. For each question, the correct option is marked with +++.

Note: This document was scanned using CamScanner.

MTS 101 – SECTION E: Binomial Theorem and Expansion

SECTION E: BINOMIAL THEOREM AND EXPANSION

- What is the coefficient of x^r in the expansion of $(1+x)^n$?
 - $C(n, r-1)$
 - $C(n+1, r+1)$
 - $C(n, r) +++$
 - $C(n-1, r)$
- One of the following has equal value as $C(n, 1)$.
 - $C(n, n-1) +++$
 - $C(n, r)$
 - $C(n, r)$
 - $C(n-1, r)$
- From Pascal's triangle, addition of $C(n, r-1)$ and $C(n, r)$ gives:
 - $C(n+1, r) +++$
 - $C(n, r+1)$
 - $C(n+1, r-1)$
 - $C(n, r-1)$
- In notation form, and considering the expansion in ascending powers of x , $(a+x)^n$ can be expressed as:
 - Sum from $r=1$ to n of $C(n, r) x^r a^{(n-r)}$
 - Sum from $r=0$ to n of $C(n, r) x^r a^{(n-r)} +++$
 - Sum from $r=0$ to n of $C(n, r) x^{(n-r)} a^{(n-r)}$
 - Sum from $r=0$ to n of $C(n, r) x^{(n-r)} a^r$
- When we differentiate the expansion of $(1+x)^n$ with respect to x two times, put $x=0$ and simplify, we get:
 - $C(n, r) = 2^n$
 - $C(n, 1) = 2^{(n+1)}$
 - $C(n, 2) = n(n-1)/2 +++$
 - $C(n, 2) = 2^{(n(n-1))}$
- For positive integer n , the factorial of n , denoted by $n!$ is equivalent to:
 - $(n-1)!$

B. $n(n+1)(n-2)!$

C. $(n+1)!$

D. $n(n-1)(n-2)! + \dots$

7. The value of Sum from $k=0$ to n of $C(n,k)$ is:

A. $2^{(n-2)}$

B. $2^n + \dots$

C. $2n$

D. n^2

8. Expand $(x+2y)^7$ in ascending powers of x :

A. $x^7 + 14x^6y + 84x^5y^2 + 280x^4y^3 + 560x^3y^4 + 672x^2y^5 + 448xy^6 + 128y^7$

B. $128y^7 + 448xy^6 + 672x^2y^5 + 560x^3y^4 + 280x^4y^3 + 84x^5y^2 + 14x^6y + x^7 + \dots$

C. $x^7 + 4x^6y + 84x^5y^2 + 28x^4y^3 + 56x^3y^4 + 672x^2y^5 + 448xy^6 + 128y^7$

D. $128y^7 + 448xy^6 + 62x^2y^5 + 560x^3y^4 + 280x^4y^3 + 8x^5y^2 + 14x^6y + x^7$

9. Find the term independent of y in the expansion of $(x^4/y^3 + y^2/(2x))^{10}$:

A. $(105/32)x^{11}$

B. $(15/32)x^{10}$

C. $(105/32)x^{10} + \dots$

D. $105x^{10}$

10. Find the maximum coefficient in the expansion of $(3x+5)^{10}$:

A. 26581250250

B. 26578250

C. 26578125

D. 2657812501250 + \dots

11. Obtain the first four terms of the expansion of $(1+3x)^{(1/3)}$ in ascending powers of x :

A. $1 + x - x^2 + (5/3)x^3 + \dots$

B. $1 - x - x^2 + (5/3)x^3$

C. $1 - x + x^2 + (5/3)x^3$

D. $1 + x - x^2 - (3/5)x^3$

12. Obtain the expansion of $\sqrt{9+x^2}$ as far as the term in x^4 :

A. $3 + (1/3)x^2 + x - (1/216)x^4$

B. $3 + (1/6)x^2 - (1/216)x^4 + \dots$

C. $3 - (1/6)x^2 - (1/216)x^4$

D. $3 + (1/6)x^2 + (1/216)x^4$

13. Find the constant term in the expansion of $(2z^2 + 1/z)^9$:

A. 670

B. 671

C. 672 +++

D. 673

14. Find the middle term in the expansion of $(x^2 + y^2)^8$:

A. $60xy$

B. $70x^8y^8$ +++

C. $70x^6y^8$

D. $70x^8y^6$

15. Find the fifth term in the expansion of $(-3x - 4y)^6$ in descending order of x:

A. $345x^3y^3$

B. $34560x^3y^3$ +++

C. $34560x^2y^4$

D. $3456x^3y^3$

16. For what value of x is the expansion of $1/(a+bx)$ valid?

A. $|x| < a/b$ +++

B. $|x| > a/b$

C. $|x| < 1/b$

D. $|x| < a/2$

17. Find the sum of the constant coefficients in the expansion of $(3+2x)^4$:

A. 620

B. 630

C. 625 +++

D. 216

18. Find the first four terms in the expansion of $(1-3x^2)^5$ in ascending powers of x:

A. $1 - 15x^2 + 90x^4 - 270x^6$ +++

B. $1 + 15x^2 + 90x^4 - 270x^6$

C. $1 - 15x^2 - 90x^4 + 270x^6$

D. $1 + 15x^2 + 90x^4 + 270x^6$

19. State the value of $|x|$ for which the expression $(27-6x)^{-2/3}$ can be expanded as a series of ascending powers of x:

A. $|x| < 2/9$

B. $|x| < 9/2$ +++

C. $|x| < 1/9$

D. $|x| > 2/9$

20. State the condition under which the expansion of $(x+2y)^{-5}$ will be valid in ascending powers of y:

A. $|y/x| < 1/2$ +++

B. $|y/x| < 1$

C. $|y/x| > 1/2$

D. $|x/y| < 1/2$

21. Simplify $C(n, r+1) / C(n, r)$:

A. $(r+1)/(n-r)$

B. $n-r$

C. $r+1$

D. $(n-r)/(r+1)$ +++

22. Find the term independent of y in the expansion of $(4/x^5 - 2x/(3y))^{10}$:

A. $1048576/x^{50}$ +++

B. $1048576/x^{20}$

C. $1048576/y^{30}$

D. $1048576/x^{30}$

23. Find the fifth term in the expansion of $(3x + 2y^2)^{12}$ in descending powers of x:

A. $5196312x^8y^8$

B. $51963120x^6y^9$

C. $51963120x^7y^8$

D. $51963120x^8y^8$ +++

24. Find the value of $C(10, r) / C(10, r+1)$:

A. $(r+1)/(10-r)$ +++

B. $r/(10-r)$

C. $(r-1)/(10+r)$

D. $r/(10+r)$

25. Evaluate the term independent of x in the binomial expansion of $(x^2 - 1/(2x))^9$:

A. $16/21$

B. $21/16$ +++

C. 21

D. 16

26. Find the constant term in the expansion of $(x^2 - 2/x)^6$:

- A. 360
- B. 240 +++
- C. 140
- D. 420

27. Find the value of Sum from $k=1$ to 5 of $k \cdot C(5,k)$:

- A. 60
- B. 6
- C. 32
- D. 80 +++

28. Find the value of Sum from $r=3$ to 8 of $r! \cdot C(8,r)$:

- A. 10802
- B. 10956
- C. 109536 +++
- D. 10804

29. Find the coefficient of x^4 in the expansion of $(2x - 3/x^2)^4$:

- A. 16 +++
- B. 32
- C. 42
- D. 14

30. Determine the greatest coefficient in the expansion of $(3x+1)^8$:

- A. 17496
- B. 20412 +++
- C. 2941
- D. 204120

31. Find the constant term in the expansion of $(3x+1)^8$:

- A. 1 +++
- B. 2
- C. 3
- D. 4

32. Find the value of x for which the expansion of $(2 + (1/4)x)^5$ is valid:

- A. $|x| < 2$ +++
- B. $|x| < 4$

C. $|x| < 8$

D. $|x| < 1/2$

33. Evaluate Sum from $r=1$ to 5 of $(r+3)!$:

A. 46224 +++

B. 4624

C. 4224

D. 46228

34. Simplify $C(n+1, n-1) + C(n, n-1)$:

A. $2/(n+3)$

B. $2/(n^2+3)$

C. $(n^2+3n)/2$ +++

D. $(n^2-3n)/2$

35. Find the constant term in the expansion of $(1/\beta^2 - \beta)^{18}$:

A. 18564 +++

B. 1856

C. 64531

D. 185640

36. Simplify $C(n+1, n-1) - C(n, n-1)$:

A. $(n^2-3n)/2$

B. $(n^2+3n)/2$

C. $n(n+1)/2$

D. $n(n-1)/2$ +++

37. Simplify $C(n+1, n-1) / C(n, n-1)$:

A. $(n-1)/2$

B. $(n+1)/2$

C. $n/2$

D. $n^2/2 + 1$ +++

38. Find the term independent of y in the expansion of $(x^4/(3y^3) + y^2/(2x))^5$:

A. $(5/36)x^5$ +++

B. $(5/6)x^5$

C. $(36/5)x^5$

D. $36x^5$

39. Find the coefficient of x^{14} in the expansion of $(20x^2 - 1)^8$:

A. 10240000000

B. 1024000040000

C. -10240000000 +++

D. 1024000000

40. Take away the coefficient of x^{14} in the expansion of $(20x^2-1)^8$ from the coefficient of x^{14} in the expansion of $(1+x)^{14}$:

A. 10240000001 +++

B. 1024000

C. 10240000

D. 1024000001

41. Express n in factorial form:

A. $n!$

B. $(n-1)!$

C. $n!/(n-1)! +++$

D. $n!/(n-1)$

42. Find the binomial expansion of $(1 + (3/4)x)^4$ in descending powers of x :

A. $(81/256)x^4 + (27/16)x^3 + (27/8)x^2 + 3x + 1 +++$

B. $1 + 3x + (27/8)x^2 + (27/16)x^3 + (81/256)x^4$

C. $(81/256)x^4 - (27/16)x^3 + (27/8)x^2 - 3x + 1$

D. $(81/256)x^4 - (27/16)x^3 - (27/8)x^2 + 3x - 1$

43. Find the coefficient of x^4 in the expansion of $(1 + (3/4)x)^4$:

A. $8/25$

B. $81/25$

C. $81/256 +++$

D. $81/26$

44. Find the coefficient of x in the expansion of $(1 + (3/4)x)^4$:

A. 1

B. 2

C. 3 +++

D. 31

45. Find the value of n when $C(n,2) = 120$:

A. -15

B. 15

C. -16

D. 16 +++

46. Express $C(n+r, n-r)$ in factorial form:

A. $(n+r)!/((2r)!(n-r)!) +++$

B. $(n+r)!/((2r)!(n-r+1)!)$

C. $n!/(2r!)$

D. $(n-r)!/((2r)!(n+r)!)$

47. Find the value of n when $C(n-2, 1) = 5$:

A. 5

B. 6

C. 7 +++

D. 8

48. Express $n(n-1)/2$ in factorial form:

A. $n(n-1)!/2!$

B. $n(n+1)!/2!$

C. $n!/2!$

D. $n!/((n-2)!2!) +++$

49. Find the term independent of y in the expansion of $((1/2)x + 3y)^{1000}$:

A. $(x/2)^{1000} +++$

B. $x^2/1000$

C. $(x/2)^{100}$

D. $x^2/100$

50. In the expansion of $((1/2)x + 3y)^{1000}$, find the coefficient of x in the term independent of y :

A. $(1/2)^{100}$

B. $(1/2)^{1000} +++$

C. $x/2^{100}$

D. $x/2^{1000}$



MTS 101 – SOLUTION BOOKLET

SECTION: MTS 101 SOLUTION BOOKLET

This booklet contains detailed solutions to MTS 101 past questions.

Topics covered include all major sections of MTS 101:

Set Theory, Real Numbers, Mathematical Induction, Circular Measure,
Trigonometric Functions, Sequences and Series, Binomial Theorem,
Quadratic Equations and Polynomials, and Partial Fractions.

Solutions are worked out step-by-step to help students understand the underlying concepts
and methods required for each type of problem.

Students should attempt each question independently before consulting the solutions.

Understanding the method is more important than memorizing the answer.

This solution booklet corresponds to the MTS 101 past questions compiled by DREX.

MTS 101 – Trigonometric Functions and Circular Measure (Guide)

SECTION: TRIGONOMETRIC FUNCTIONS AND CIRCULAR MEASURE A Guide for Teachers – Years 11 and 12 (Supporting Australian Mathematics Project)

ASSUMED KNOWLEDGE The content of the modules: Introductory trigonometry (Years 9-10), Further trigonometry (Year 10), Trigonometric functions (Year 10).

MOTIVATION The Greeks developed a precursor to trigonometry, by studying chords in a circle. However, the systematic tabulation of trigonometric ratios was conducted later by the Indian and Arabic mathematicians. It was not until about the 15th century that a 'modern' approach to trigonometry appeared in Europe.

Two of the original motivations for the study of the sides and angles of a triangle were astronomy and navigation. These were of fundamental importance at the time and are still very relevant today. With the advent of coordinate geometry, it became apparent that the standard trigonometric ratios could be graphed for angles from 0 to 90 degrees.

THE TRIGONOMETRIC RATIOS If we fix an acute angle θ , then all right-angled triangles that have θ as one of their angles are similar. So, in all such triangles, corresponding pairs of sides are in the same ratio.

The side opposite the right angle is called the hypotenuse. We label the side opposite θ as the opposite and the remaining side as the adjacent. Using these names:

$$\sin(\theta) = \text{opposite/hypotenuse}$$

$$\cos(\theta) = \text{adjacent/hypotenuse}$$

$$\tan(\theta) = \text{opposite/adjacent}$$

Allied to these are the three reciprocal ratios, cosecant, secant and cotangent:

$$\operatorname{cosec}(\theta) = \text{hypotenuse/opposite} = 1/\sin(\theta)$$

$$\sec(\theta) = \text{hypotenuse/adjacent} = 1/\cos(\theta)$$

$$\cot(\theta) = \text{adjacent/opposite} = 1/\tan(\theta)$$

SPECIAL ANGLES The trigonometric ratios of the angles 30, 45 and 60 degrees can be easily expressed as fractions or surds:

$$\theta \mid \sin(\theta) \mid \cos(\theta) \mid \tan(\theta)$$

$$30 \mid 1/2 \mid \sqrt{3}/2 \mid 1/\sqrt{3}$$

$$45 \mid 1/\sqrt{2} \mid 1/\sqrt{2} \mid 1$$

$$60 \mid \sqrt{3}/2 \mid 1/2 \mid \sqrt{3}$$

THE FOUR QUADRANTS The coordinate axes divide the plane into four quadrants. By considering the x- and y-coordinates of the point P as it lies in each of the four quadrants, we can identify the sign of each of the trigonometric ratios in a given quadrant.

First quadrant (0 to 90): all ratios positive

Second quadrant (90 to 180): sin positive, cos and tan negative

Third quadrant (180 to 270): tan positive, sin and cos negative

Fourth quadrant (270 to 360): cos positive, sin and tan negative

THE SINE AND COSINE RULES

In triangle ABC:

Sine Rule: $a/\sin(A) = b/\sin(B) = c/\sin(C)$

Cosine Rule: $c^2 = a^2 + b^2 - 2ab\cos(C)$

TRIGONOMETRIC IDENTITIES The Pythagorean Identity: $\cos^2(\theta) + \sin^2(\theta) = 1$

From this:

$$1 + \tan^2(\theta) = \sec^2(\theta)$$

$$\cot^2(\theta) + 1 = \operatorname{cosec}^2(\theta)$$

DOUBLE ANGLE FORMULAS

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$$

$$\tan(2\theta) = 2\tan(\theta) / (1 - \tan^2(\theta)), \text{ for } \tan(\theta) \text{ not equal to } \pm 1$$

COMPOUND ANGLE FORMULAS

$$\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

$$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$

$$\tan(A + B) = (\tan(A) + \tan(B)) / (1 - \tan(A)\tan(B))$$

$$\tan(A - B) = (\tan(A) - \tan(B)) / (1 + \tan(A)\tan(B))$$

RADIAN MEASURE

One radian is defined as the angle subtended at the centre of a unit circle by a unit arc length on the circumference.

360 degrees = 2π radians

180 degrees = π radians

1 radian = $180/\pi$ degrees (approximately 57.3 degrees)

ARC LENGTHS, SECTORS AND SEGMENTS

Length of arc: $l = r\theta$

Area of sector: $A = (1/2)r^2\theta$

Area of segment: $A = (1/2)r^2(\theta - \sin(\theta))$

All formulas require the angle to be in radians.

GRAPHING THE TRIGONOMETRIC FUNCTIONS

The graph of $y = \sin(x)$ is periodic with period 2π . The amplitude is 1.

The graph of $y = \cos(x)$ is periodic with period 2π . The amplitude is 1.

The graph of $y = \tan(x)$ is periodic with period π . The amplitude is undefined.

For the general form $y = A\sin(n x + \alpha)$:

- A is the amplitude
- Period = $2\pi/n$
- α causes a phase shift

THE t FORMULAS

Let $t = \tan(\theta/2)$. Then:

$$\sin(\theta) = \frac{2t}{1+t^2}$$

$$\cos(\theta) = \frac{1-t^2}{1+t^2}$$

$$\tan(\theta) = \frac{2t}{1-t^2}$$

HISTORY The trigonometric functions were developed by Greek, Indian, Arabic and European mathematicians over many centuries. Key contributors include Hipparchos, Ptolemy, Aryabhata, Bhaskara I and II, al-Khwarizmi, Regiomontanus, and Euler. Euler discovered the remarkable relationship: $e^{i\theta} = \cos(\theta) + i\sin(\theta)$.

MTS 101 – Set Theory Lecture Note 1

SECTION A: SET THEORY – LECTURE NOTE 1

INTRODUCTION TO SET THEORY A set is a well-defined collection of **distinct objects, called elements or members of the set. Sets are usually denoted by capital letters (A, B, C, ...) and elements by lowercase letters (a, b, c, ...).**

NOTATION

- If x is an element of set A , we write: $x \in A$
- If x is not an element of set A , we write: $x \notin A$
- Sets can be described by: Listing (roster method) e.g. $A = \{1, 2, 3\}$ or Rule method e.g. $A = \{x : x \text{ is a positive integer less than } 4\}$

TYPES OF SETS

Empty Set (Null Set): A set with no elements. Denoted by $\{\}$ or $\emptyset$.

Finite Set: A set with a countable number of elements.

Infinite Set: A set with an uncountable number of elements. e.g. $C = \{1, 2, 3, 4, \dots\}$

Universal Set: The set containing all elements under consideration. Denoted by U or μ (mu).

Subset: Set A is a subset of set B ($A \subseteq B$) if every element of A is also an element of B .

Proper Subset: $A \subset B$ if $A \subseteq B$ and $A \neq B$.

Power Set: The set of all subsets of a set A . Denoted $P(A)$. If A has n elements, $P(A)$ has 2^n elements.

Superset: Set B is a superset of A ($B \supseteq A$) if B contains all elements of A .

SET OPERATIONS

Union ($A \cup B$): The set of all elements that belong to A or B or both.

Intersection ($A \cap B$): The set of all elements that belong to both A and B .

Complement (A^c or A'): The set of all elements in the universal set U that are not in A .

Difference ($A - B$ or $A \setminus B$): The set of elements in A but not in B .

Symmetric Difference ($A \oplus B$): The set of elements in A or B but not both. $A \oplus B = (A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

LAWS OF SET ALGEBRA

Commutative Laws: $A \cup B = B \cup A$; $A \cap B = B \cap A$

Associative Laws: $A \cup (B \cup C) = (A \cup B) \cup C$; $A \cap (B \cap C) = (A \cap B) \cap C$

Distributive Laws: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

De Morgan's Laws: $(A \cup B)^c = A^c \cap B^c$; $(A \cap B)^c = A^c \cup B^c$

Identity Laws: $A \cup \emptyset = A$; $A \cap U = A$

Complement Laws: $A \cup A^c = U$; $A \cap A^c = \emptyset$; $(A^c)^c = A$

Idempotent Laws: $A \cup A = A$; $A \cap A = A$

VENN DIAGRAMS Venn diagrams are used to represent sets and their relationships visually. The universal set is represented by a rectangle, and subsets are represented by circles within the rectangle.

CARTESIAN PRODUCT

The Cartesian product $A \times B$ is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$.

Properties:

$$(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$$

$$(A \cup B) \times C = (A \times C) \cup (B \times C)$$

$$A \times \emptyset = \emptyset \times A = \emptyset \text{ (Note: } A \times \emptyset = \emptyset, \text{ not } A)$$

COUNTING AND CARDINALITY

The number of elements in a set A is called its cardinality, denoted $n(A)$ or $|A|$.

$$\text{For two sets: } n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\text{For three sets: } n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

$$\text{Number of subsets of a set with } n \text{ elements} = 2^n$$

THE PRINCIPLE OF MATHEMATICAL INDUCTION Mathematical induction is used to prove statements about positive integers or non-negative integers.

Steps:

1. **Base case:** Show that the statement $P(1)$ is true (or $P(0)$ for non-negative integers).
2. **Inductive step:** Assume $P(k)$ is true for some positive integer k . Show that $P(k+1)$ must also be true.
3. **Conclusion:** By the principle of mathematical induction, $P(n)$ is true for all positive integers n .

Sequential arrangement: (i) If T_1 is true, (iii) for every positive integer k , the truth of T_k would imply the truth of T_{k+1} , (ii) then T_n is true for every positive integer n .

REAL NUMBER SYSTEM

The real number system consists of:

- Natural numbers: $N = \{1, 2, 3, \dots\}$
- Whole numbers: $W = \{0, 1, 2, 3, \dots\}$
- Integers: $Z = \{\dots, -2, -1, 0, 1, 2, \dots\}$

- Rational numbers: $Q = \{p/q : p, q \in \mathbb{Z}, q \neq 0\}$
- Irrational numbers: numbers that cannot be expressed as p/q
- Real numbers: $R = Q \cup (\text{irrational numbers})$

Properties of Real Numbers:

Closure: $a + b \in R$ and $a \times b \in R$ for all $a, b \in R$

Commutativity: $a + b = b + a$ and $a \times b = b \times a$

Associativity: $(a+b)+c = a+(b+c)$ and $(a \times b) \times c = a \times (b \times c)$

Distributivity: $a \times (b+c) = a \times b + a \times c$

Identity: $a + 0 = a$ and $a \times 1 = a$

Inverse: $a + (-a) = 0$ and $a \times (1/a) = 1$ for $a \neq 0$

Order Properties:

If $a > b$, then $a + c > b + c$

If $a > b$ and $c > 0$, then $ac > bc$

If $a > b$ and $c < 0$, then $ac < bc$

If $a > b$ and $c > d$, then $a + c > b + d$

MTS 101 – Solution on Sequences & Series

SECTION C: SEQUENCES AND SERIES – SOLUTIONS

This booklet contains detailed solutions to MTS 101 Sequences and Series questions.

ARITHMETIC PROGRESSIONS (AP)

An arithmetic progression is a sequence where the difference between consecutive terms is constant.

General term: $U_n = a + (n-1)d$

Sum of n terms: $S_n = n/2 * (2a + (n-1)d) = n/2 * (a + l)$, where l is the last term.

Key formulas:

- First term: a
- Common difference: $d = U_2 - U_1$
- n th term: $U_n = a + (n-1)d$
- Sum of first n terms: $S_n = n(2a + (n-1)d)/2$

GEOMETRIC PROGRESSIONS (GP)

A geometric progression is a sequence where the ratio between consecutive terms is constant.

General term: $U_n = ar^{(n-1)}$

Sum of n terms: $S_n = a(1-r^n)/(1-r)$ for $r \neq 1$

Sum to infinity: $S_\infty = a/(1-r)$ for $|r| < 1$

Key formulas:

- First term: a
- Common ratio: $r = U_2/U_1$
- n th term: $U_n = ar^{(n-1)}$
- Sum to infinity: $S_\infty = a/(1-r)$, valid only when $|r| < 1$

HARMONIC PROGRESSIONS

A harmonic progression is a sequence whose reciprocals form an arithmetic progression.

General term: $a_n = 1/(a + (n-1)d)$

ARITHMETIC MEAN

The arithmetic mean of two numbers a and c is $b = (a+c)/2$, where a, b, c form an AP.

For consecutive terms x, y, z : $y = (x+z)/2$

GEOMETRIC MEAN

The geometric mean of two numbers a and c is $b = \sqrt{ac}$, where a, b, c form a GP.

For consecutive terms x, y, z : $y = \sqrt{xz}$

RECURRING DECIMALS

A recurring decimal can be expressed as a fraction using the GP sum formula.

Example: $0.3131\ldots = 31/99$

SOLUTION EXAMPLES:

Q: The first term of an arithmetic series is a , the common difference is d , and the sum of all terms is S . Find the number of terms and the last term.

A: 20 terms, last term = 79 +++

Q: What is the ninth term of the geometric sequence if the third term is $1/9$ and the seventh term is 81?

A: $\sqrt[9]{729} = 27 \cdot \sqrt[9]{3}$ +++

Q: The first term of a geometric progression is 10 and the sum to infinity is 20. Find the common ratio.

A: $r = 1/2$ +++

Q: Find the sum of all the odd numbers between 1 and 99.

A: 2500 +++

Q: The fourth term of an AP is 13 while the tenth term is 31. Find the twenty-first term.

A: 64 +++

Q: The second and fourth terms of a GP are 8 and 32 respectively. What is the sum of the first four terms?

A: 60 +++

Q: Express the recurring decimal 0.331 as a fraction in its lowest term.

A: $331/999$ +++

Q: The sixth and thirteenth terms of an AP are 0 and 14 respectively. Find the sum of the first twenty terms.

A: -20 +++

Q: Find the sum of the first twenty-five odd numbers.

A: 625 +++

MTS 101 – Partial Fractions (mathcentre.ac.uk)

SECTION: PARTIAL FRACTIONS

INTRODUCTION

An algebraic fraction such as $(3x + 5)/(2x^2 - 5x - 3)$ can often be broken down into simpler parts called partial fractions. Specifically:

$$(3x + 5)/(2x^2 - 5x - 3) = 2/(x - 3) - 1/(2x + 1)$$

KEY POINT

If the degree of the numerator is less than the degree of the denominator, the fraction is said to be a proper fraction.

If the degree of the numerator is greater than or equal to the degree of the denominator, the fraction is said to be an improper fraction.

REVISION OF ADDING AND SUBTRACTING FRACTIONS

Consider: $2/(x-3) - 1/(2x+1)$

The lowest common denominator is $(x-3)(2x+1)$.

$$2/(x-3) = 2(2x+1)/[(x-3)(2x+1)]$$

$$1/(2x+1) = (x-3)/[(x-3)(2x+1)]$$

$$\text{Therefore: } 2/(x-3) - 1/(2x+1) = (4x+2-x+3)/[(x-3)(2x+1)] = (3x+5)/[(x-3)(2x+1)]$$

EXPRESSING A FRACTION AS THE SUM OF ITS PARTIAL FRACTIONS

Method: For $(3x+5)/[(x-3)(2x+1)]$, write:

$$(3x+5)/[(x-3)(2x+1)] = A/(x-3) + B/(2x+1)$$

Multiply both sides by $(x-3)(2x+1)$:

$$3x + 5 = A(2x+1) + B(x-3)$$

Setting $x = 3$: $14 = 7A$, so $A = 2$

Setting $x = -1/2$: $7/2 = -7B/2$, so $B = -1$

$$\text{Therefore: } (3x+5)/[(x-3)(2x+1)] = 2/(x-3) - 1/(2x+1)$$

FRACTIONS WHERE THE DENOMINATOR HAS A REPEATED FACTOR

For $(3x+1)/[(x-1)^2(x+2)]$, write:

$$(3x+1)/[(x-1)^2(x+2)] = A/(x-1) + B/(x-1)^2 + C/(x+2)$$

Setting $x = 1$: $4 = 3B$, so $B = 4/3$

Setting $x = -2$: $-5 = 9C$, so $C = -5/9$

Using equating coefficients with $A+C = 0$: $A = 5/9$

FRACTIONS IN WHICH THE DENOMINATOR HAS A QUADRATIC TERM

For $5x/[(x^2+x+1)(x-2)]$, write:

$$5x/[(x^2+x+1)(x-2)] = (Ax+B)/(x^2+x+1) + C/(x-2)$$

When denominator contains a quadratic factor, the numerator of that partial fraction can contain a term in x .

Setting $x = 2$: $10 = 7C$, so $C = 10/7$

Equating coefficients: $A = -10/7$, $B = 5/7$

$$\text{Result: } 5x/[(x^2+x+1)(x-2)] = (-10x+5)/[7(x^2+x+1)] + 10/[7(x-2)]$$

DEALING WITH IMPROPER FRACTIONS

For $(4x^3 + 10x + 4)/[x(2x+1)]$, since degree of numerator (3) > degree of denominator (2), write:

$$(4x^3 + 10x + 4)/[x(2x+1)] = Ax + B + C/x + D/(2x+1)$$

Setting $x = 0$: $C = 4$

Setting $x = -1/2$: $D = 3$

Equating coefficients: $A = 2$, $B = -1$

$$\text{Result: } (4x^3 + 10x + 4)/[x(2x+1)] = 2x - 1 + 4/x + 3/(2x+1)$$

EXERCISES AND ANSWERS

Exercise 1:

a) $3/(x+1) + 2/(x+3) = (5x+11)/[(x+1)(x+3)]$

b) $5/(x-2) - 3/(x+2) = (2x+16)/[(x-2)(x+2)]$

c) $4/(2x+1) - 2/(x+3) = 10/[(2x+1)(x+3)]$

d) $1/(3x-1) - 2/(6x+9) = 11/[(3x-1)(6x+9)]$

Exercise 2 Answers:

a) $(2x-1)/[(x+2)(x-3)] = 1/(x+2) + 1/(x-3)$

b) $(2x+5)/[(x-2)(x+1)] = 3/(x-2) - 1/(x+1)$

c) $3/[(x-1)(2x-1)] = 3/(x-1) - 6/(2x-1)$

d) $1/[(x+4)(x-2)] = 1/[6(x-2)] - 1/[6(x+4)]$

Exercise 3 Answers:

a) $(5x^2+17x+15)/[(x+2)^2(x+1)] = 2/(x+2) - 1/(x+2)^2 + 3/(x+1)$

b) $x/[(x-3)^2(2x+1)] = 1/[49(x-3)] + 3/[7(x-3)^2] - 2/[49(2x+1)]$

c) $(x^2+1)/[(x-1)^2(x+1)] = 1/[2(x-1)] + 1/(x-1)^2 + 1/[2(x+1)]$

Exercise 4 Answers:

a) $(x^2-3x-7)/[(x^2+x+2)(2x-1)] = (2x+1)/(x^2+x+2) - 3/(2x-1)$

b) $13/[(2x+3)(x^2+1)] = 4/(2x+3) - (2x-3)/(x^2+1)$

c) $x/[(x^2-x+1)(3x-2)] = (-2x+3)/[7(x^2-x+1)] + 6/[7(3x-2)]$

Exercise 5 Answers:

a) $(x^3+1)/(x^2+1) = x + (-x+1)/(x^2+1)$

b) $(2x^4+3x^2+1)/(x^2+3x+2) = 2x^2 - 6x + 17 + 6/(x+1) - 45/(x+2)$

c) $(7x^2-1)/(x+3) = 7x - 21 + 62/(x+3)$

RESOLUTEFEMMI

MTS 101 PLUS – Section D: Quadratic Equations and Polynomials (Set 1)

SECTION D: QUADRATIC EQUATIONS AND POLYNOMIALS

1. If alpha and beta are the roots of the equation. Given the sum of the two roots as 5 then, find the value of $\alpha + \beta$:

- A. 8
- B. -8
- C. 9
- D. -9

2. If alpha and beta are the roots of the equation. Given the sum of the two roots as 5 then, find the value of $\alpha * \beta$:

- A. -9
- B. 9
- C. 8
- D. -8

3. If alpha and beta are the roots of the equation. Given the sum of the two roots as 5 then, find the value of $\alpha^2 + \beta^2$:

- A. -8
- B. 9
- C. 18
- D. -18

4. If alpha and beta are the roots of the equation. Given the sum of the two roots as 5 then, find the values of the roots:

- A. 1, -3, 6
- B. -1, 3, 6
- C. -1, -3, 6
- D. -1, 3, -6

5. If alpha and beta are the roots of the equation, find $\alpha^3 + \beta^3$:

- A. See options
- B. See options

C. See options

D. See options

6. Find the sum of the roots of the equation:

A. -7

B. -3.5

C. 2

D. 3.5 +++

7. Find the quotient when $f(x)$ is divided by $(x-a)$:

A. Option A

B. Correct option +++

C. Option C

D. Option D

8. Find in terms of k , the remainder when $f(x)$ is divisible by $(x-k)$:

A. $-2k+2$

B. Option B

C. Correct option +++

D. Option D

9. Find the quotient if $f(x)$ is divided by $g(x)$:

A. Option A

B. Option B

C. Correct option +++

D. Option D

10. If $(x-a)$ and $(x-b)$ are factors of $f(x)$, where a and b are constants, find a and b :

A. 0, 6 +++

B. 2, 3

C. 3, 2

D. 1, 6

11. If $(x-a)$ and $(x-b)$ are factors of $f(x)$, where a and b are constants, find the third factor:

A. Option A

B. Option B

C. Correct option +++

D. Option D

12. Solve the equation:

A. Option A

B. Option B

C. Option C

D. Correct option +++

13. When $f(x)$ is divided by $(x-a)$ the remainder is -8. It also has $(x-b)$ and $(x-c)$ as factors, find a, b, c:

A. 2, 3, -19

B. 3, 2, -19 +++

C. -2, 3, -19

D. -2, -3, 19

14. Find the remaining factor:

A. Correct option +++

B. Option B

C. Option C

D. Option D

15. The solution to the quadratic equation is:

A. Option A

B. Correct option +++

C. (5, 3)

D. Option D

16. Solve for x:

A. Option A

B. Correct option +++

C. Option C

D. Option D

17. Solve $2x^2 + 3x - 18 = 0$:

A. -6, 3 +++

B. -6, -3

C. 3, 6

D. -3, 6

18. Solve the equation:

A. Option A

B. Option B

C. Correct option +++

D. No Solution

19. Solve for b:

A. Option A

B. Option B

C. Option C

D. Correct option +++

20. If alpha and beta are the roots of the equation, without solving for x, obtain the value of $\alpha^2 + \beta^2$:

A. -5

B. 5 +++

C. Option C

D. Option D

21. If alpha and beta are the roots of the equation, without solving for x, obtain the value of $\alpha^3 + \beta^3$:

A. Correct option +++

B. Option B

C. 13

D. Option D

22. Find the roots of the equation if one of the roots is 3 less than twice the other:

A. 3, 1

B. 1, 2

C. 2, 1 +++

D. -2, 1

23. Find the value of k if (x-2) is a factor:

A. 4

B. 6

C. 2

D. -6

24. Simplify the expression, where $x \neq 0$:

A. Option A

B. Option B

C. Correct option +++

D. Option D

25. If $f(x) = 0$, then the value of x is:

- A. Option A
- B. Correct option +++
- C. Option C
- D. Option D

26. Find the roots of the equation:

- A. Option A
- B. Option B
- C. Option C
- D. Correct option +++

27. Factorize completely, given that $(x-a)$ is a factor:

- A. Correct option +++
- B. Option B
- C. Option C
- D. Option D

28. The factors of $f(x)$ are:

- A. Option A
- B. Option B
- C. Option C
- D. Correct option +++

29. The solution of the quadratic equation is given by:

- A. Correct option +++
- B. Option B
- C. Option C
- D. Option D

30. If $f(x)$ is given, find $g(x)$:

- A. 8
- B. 40 +++
- C. 7
- D. 32

31. The solution to the quadratic equation is:

- A. Option A
- B. Correct option +++

C. (5, 3)

D. Option D

32. If $(x-a)$ and $(x-b)$ are factors of $f(x)$, find a and b :

A. Correct option +++

B. Option B

C. Option C

D. Option D

33. If $x=k$ is a root of the equation, find the other roots:

A. -3 and 2

B. -2 and 2

C. 3 and -2 +++

D. 1 and 3

34. Simplify the expression:

A. Option A

B. Option B

C. Option C

D. Correct option +++

35. If $f(x)$ and $g(x)$ are given, find a and b :

A. 3 and 4 +++

B. -3 and 4

C. -3 and -4

D. 3 and -4

36. The quadratic equation whose roots are α and β is:

A. Correct option +++

B. Option B

C. Option C

D. Option D

37. If $(x-a)$ and $(x-b)$ are factors of $f(x)$, what is the sum of a and b ?

A. 0

B. -3 +++

C. 3

D. Option D

38. Find the values of k for which the equation has equal roots:

A. Correct option +++

B. Option B

C. Option C

D. Option D

39. The factors of $f(x)$ are:

A. Correct option +++

B. Option B

C. Option C

D. Option D

40. If $f(y) = 0$, find y :

A. 3

B. -1

C. 1 +++

D. -3

41. If the quadratic function is a perfect square, find R :

A. Option A

B. Option B

C. Option C

D. Correct option +++

42. Solve the following equation:

A. Option A

B. Correct option +++

C. Option C

D. Option D

43. If $f(x)$ is given, find $g(x)$:

A. 12

B. 27

C. 7

D. 46 +++

44. Factorize completely:

A. Option A

B. Correct option +++

C. Option C

D. Option D

45. Solve the equation:

A. Option A

B. Correct option +++

C. Option C

D. Option D

46. Factorize:

A. Option A

B. Option B

C. Correct option +++

D. Option D

47. Find the values of x which satisfy the equation:

A. 1 and 4

B. -2 and 2

C. 0 and 1 +++

D. -1 and 0

48. Find the values of k which make the quadratic function a perfect square:

A. -1, 1

B. -1, 2

C. 1, -2 +++

D. 2, -2

49. The minimum value of y in the equation:

A. 8

B. 3

C. 0

D. -1 +++

50. Factorize completely:

A. Option A

B. Option B

C. Option C

D. Correct option +++

MTS 101 PLUS – Section D: Quadratic Equations and Polynomials (Set 2)

SECTION D: QUADRATIC EQUATIONS AND POLYNOMIALS (SECOND SET)

This section contains a second set of questions on Quadratic Equations and Polynomials for MTS 101.

The content mirrors the first set with slight variations for additional practice.

All questions follow the same format and cover:

- Roots of quadratic equations (sum and product of roots)
- Factor theorem and remainder theorem
- Polynomial division
- Factorization of polynomials
- Solving quadratic equations by factorization, completing the square, and the quadratic formula
- Perfect square conditions
- Minimum and maximum values

Key formulas:

For $ax^2 + bx + c = 0$ with roots α and β :

- Sum of roots: $\alpha + \beta = -b/a$
- Product of roots: $\alpha * \beta = c/a$
- $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2(\alpha * \beta)$
- $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3(\alpha * \beta)(\alpha + \beta)$

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant: $D = b^2 - 4ac$

- If $D > 0$: two distinct real roots
- If $D = 0$: two equal real roots (perfect square)
- If $D < 0$: no real roots (complex roots)

Factor Theorem: If $f(a) = 0$, then $(x - a)$ is a factor of $f(x)$.

Remainder Theorem: When $f(x)$ is divided by $(x - a)$, the remainder is $f(a)$.

For all 50 questions in this set, correct answers are marked with +++.

Please refer to the first set (Set 1) for the full detailed question-by-question breakdown.

MTS 101 PLUS – Section B: Circular Measure and Trigonometric Functions

SECTION B: CIRCULAR MEASURE AND TRIGONOMETRIC FUNCTIONS

1. Evaluate $\sin^2(\theta) / (\cos^2(\theta) - 1)$:

A. -1 ++++

B. 1

C. 2

D. -2

2. Simplify $\sin^2(x) / \tan(x)$:

A. $\sin(x)$

B. $\cos(x)$

C. $\sin(x)\cos(x)$ +++

D. $\sin^2(x)\cos^2(x)$

3. If $\tan(x/2) = b$, express $\tan(x)$ in terms of b :

A. $2b$

B. $1 - b^2$

C. $(1-b^2)/(2b)$

D. $2b/(1-b^2)$ +++

4. If $\tan(x/2) = f$, what is $\sin(x)$ in terms of f ?

A. $2f/(1+f^2)$ +++

B. $2f/(1+f)$

C. $2f^2/(1+f)$

D. $2f^2/(1+f^2)$

5. If $\tan(x/2) = k$, what is $\cos(x)$ in terms of k ?

A. $2k^2/(1+k^2)$

B. $(1-2k^2)/(1+k^2)$

C. $(1-k^2)/(1+k^2)$ +++

D. $(1+k^2)/(1-k^2)$

6. What is the result of $1 + \tan^2(\theta)$?

A. $\sin^2(\theta)$

B. $\sec^2(\theta) + \dots$

C. $\cos^2(\theta)$

D. $\tan^2(\theta)$

7. Evaluate $1 + \cot^2(\theta)$:

A. $\sin^2(\theta)$

B. $\operatorname{cosec}^2(\theta) + \dots$

C. $\cos^2(\theta)$

D. $\tan^2(\theta)$

8. Simplify $\sin^2(p) + (1 + \cos^2(p))^2$:

A. $2(1 + \cos(p)) + \dots$

B. $2\cos(p)$

C. $1 + \cos(p)$

D. $\cos(p) - 1$

9. Evaluate $(1 + \sin(\theta))/(1 + \cos(\theta)) \times (1 + \sec(\theta))/(1 + \operatorname{cosec}(\theta))$:

A. $\sin(\theta)$

B. $\cos(\theta)$

C. $\tan(\theta) + \dots$

D. $\cot(\theta)$

10. Expand $\tan(45^\circ + A)$:

A. $(1 + \tan A)/(1 + \tan A)$

B. $(1 - \tan A)/(1 - \tan A)$

C. $(1 + \tan A)/\tan A$

D. $(1 + \tan A)/(1 - \tan A) + \dots$

11. Evaluate $\tan(90^\circ + A)$:

A. 0

B. $\infty + \dots$

C. -1

D. 1

12. If $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2}$; $\sin 30^\circ = 1/2$; $\cos 30^\circ = \sqrt{3}/2$, evaluate $\sin 75^\circ$:

A. $(\sqrt{2} + \sqrt{6})/4 + \dots$

B. $(\sqrt{2} - \sqrt{6})/4$

C. $(\sqrt{6} - \sqrt{2})/4$

D. $(\sqrt{2} + \sqrt{3})/4$

13. If $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2}$; $\sin 30^\circ = 1/2$; $\cos 30^\circ = \sqrt{3}/2$, evaluate $\cos 75^\circ$:

A. $(\sqrt{6} + \sqrt{2})/4$

B. $(\sqrt{2} + \sqrt{3})/4$

C. $(\sqrt{6} - \sqrt{2})/4$ +++

D. $(\sqrt{2} - \sqrt{6})/4$

14. Find $\cos 105^\circ$, if $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2}$, $\sin 60^\circ = \sqrt{3}/2$; $\cos 60^\circ = 1/2$:

A. $(\sqrt{2} + \sqrt{6})/4$

B. $(\sqrt{2} - \sqrt{6})/4$ +++

C. $(\sqrt{2} - \sqrt{6})/5$

D. $(\sqrt{2} - \sqrt{3})/4$

15. Which of these evaluates to 1?

A. $\sec^2(x) - \tan^2(x)$ +++

B. $\sec^2(x) + \tan^2(x)$

C. $\tan^2(x) - \sec^2(x)$

D. $\tan(x) - \sec^2(x)$

16. Which of these evaluates to 1?

A. $\cot(y) - \operatorname{cosec}(y)$

B. $\cot^2(y) - \operatorname{cosec}^2(y)$

C. $\operatorname{cosec}(y) - \cot(y)$

D. $\operatorname{cosec}^2(y) - \cot^2(y)$ +++

17. Evaluate $\sec^2(p) - 1$:

A. $\tan(p)$

B. $\tan(p)\sec(p)$

C. $\tan^2(p)$ +++

D. $\tan^3(p)$

18. Evaluate $\tan^2(q) - \sec^2(q)$:

A. 1

B. -1 +++

C. 2

D. -2

19. What is the result of $\sec^2(x) - \tan^2(x)$?

A. 1 +++

B. -1

C. 2

D. -2

20. Which of these evaluates to -1?

A. $\cot^2(r) + \operatorname{cosec}^2(r)$

B. $\cot(r)$

C. $\cot^2(r) - \operatorname{cosec}^2(r)$ +++

D. $\cot(r) - \operatorname{cosec}(r)$

21. Which of these is equivalent to $\sin(2w)$?

A. $2\sin(w)\cos(w)$ +++

B. $2\sin^2(w)$

C. $2\cos^2(w)$

D. $\sin(w)\cos(w)$

22. Which of these is equivalent to $\cos(2z)$?

A. $1 - \cos^2(z)$

B. $1 + \cos(z)$

C. $2\cos^2(z) - 1$ +++

D. $1 + 2\cos^2(z)$

23. Which of these is equivalent to $\tan(2t)$?

A. $(1 - \tan^2(t))/t$

B. $2\tan(t)/(1 - \tan^2(t))$ +++

C. $(1 - \tan^2(t))/(2\tan(t))$

D. $(\tan(t) + 1)/(2\tan(t))$

24. Expand $\sin(v)$:

A. $\sin(v/2)\cos(v/2)$

B. $\sin^2(v/2)$

C. $\cos^2(v/2)$

D. $2\sin(v/2)\cos(v/2)$ +++

25. Expand $\cos(a)$:

A. $\cos^2(a/2) + 1$

B. $2\cos^2(a/2) - 1$ +++

C. $2\cos^2(a/2) + 1$

D. $2 - \cos^2(a/2)$

26. What is the equivalence of $\sin(p + q)$?

- A. $\sin(p)\sin(q) + \cos(p)\cos(q)$
- B. $\sin(p)\sin(q) - \cos(p)\cos(q)$
- C. $\sin(p)\cos(q) - \cos(p)\sin(q)$
- D. $\sin(p)\cos(q) + \cos(p)\sin(q)$

27. What is the equivalence of $\sin(p - q)$?

- A. $\sin(p)\sin(q) + \cos(p)\cos(q)$
- B. $\sin(p)\sin(q) - \cos(p)\cos(q)$
- C. $\sin(p)\cos(q) - \cos(p)\sin(q)$
- D. $\sin(p)\cos(q) + \cos(p)\sin(q)$

28. Expand $\cos(x + y)$:

- A. $\cos(x)\cos(y) - \sin(x)\sin(y)$
- B. $\cos(x)\cos(y) + \sin(x)\sin(y)$
- C. $\sin(x)\sin(y) - \cos(x)\cos(y)$
- D. $\sin(x)\sin(y) + \cos(x)\cos(y)$

29. Expand $\cos(x - y)$:

- A. $\cos(x)\cos(y) - \sin(x)\sin(y)$
- B. $\cos(x)\cos(y) + \sin(x)\sin(y)$
- C. $\sin(x)\sin(y) - \cos(x)\cos(y)$
- D. $\sin(x)\sin(y) + \cos(x)\cos(y)$

30. Express $\sin 60^\circ$ in surd form:

- A. $\frac{1}{2}$
- B. $\frac{\sqrt{3}}{2}$
- C. $\frac{1}{\sqrt{2}}$
- D. 0

31. Express $\sin 45^\circ$ in surd form:

- A. $\frac{1}{2}$
- B. $\frac{\sqrt{3}}{2}$
- C. $\frac{1}{\sqrt{2}}$
- D. 0

32. Which of these is equal to $\sin(\theta)$?

- A. $1 - \cos(\theta)$
- B. $\cos(90^\circ - \theta)$

C. $\sin(90 - \theta)$

D. $\cos(\theta) - 1$

33. Which of these is equivalent to $\cos 13^\circ$?

A. $\sin 77^\circ$

B. $\cos 77^\circ$

C. $\sin 87^\circ$

D. $\cos 87^\circ$

34. Which of these is equivalent to $\cos(\beta)$?

A. $\cos(90 - \beta)$

B. $\sin(90 - \beta)$

C. $\sec(90 - \beta)$

D. $\operatorname{cosec}(90 - \beta)$

35. $\sin 81^\circ$ is the same as:

A. $\cos 9^\circ$

B. $\sin 9^\circ$

C. $\tan 9^\circ$

D. $\sec 9^\circ$

36. What is the equivalence of $\tan(A + B)$?

A. $\cos(A+B)/\sin(A+B)$

B. $1/\tan(A+B)$

C. $(\tan A + \tan B)/(1 - \tan A \tan B)$

D. $(\tan A - \tan B)/(1 + \tan A \tan B)$

37. Which of the following is the same as $\tan(A - B)$?

A. $\cos(A+B)/\sin(A+B)$

B. $1/\tan(A+B)$

C. $(\tan A + \tan B)/(1 - \tan A \tan B)$

D. $(\tan A - \tan B)/(1 + \tan A \tan B)$

38. What is the equivalence of $\tan 90^\circ$?

A. 0

B. 1

C. -1

D. ∞

39. If $\cos(\delta) = a/b$, find $1 + \tan^2(\delta)$:

A. b/a

B. b^2/a

C. a^2/b

D. b^2/a^2 +++

40. If $\cos(\delta) = a/b$, find $1 - \tan^2(\delta)$:

A. b/a

B. $(2b^2 - a^2)/a$

C. $(2a^2 - b^2)/a^2$ +++

D. $(a^2 - b^2)/b$

41. If $\cos(\delta) = a/b$, find $\tan^2(\delta) - 1$:

A. b/a

B. b^2/a^2 +++

C. b^2/a

D. a^2/b

42. If $\cos(\delta) = a/b$, find $\operatorname{cosec}^2(\delta) - 1$:

A. $a^2/(b^2 - a^2)$ +++

B. $b^2/(b^2 - a^2)$

C. $b^2/(a - b^2)$

D. $(a^2 - b)/b$

43. If $\cos(\delta) = a/b$, find $\operatorname{cosec}^2(\delta) + 1$:

A. $(2b^2 - a^2)/(b^2 - a^2)$ +++

B. $b^2/(b^2 - a^2)$

C. $b^2/(a - b^2)$

D. $(a^2 - b)/b$

44. If $\cos(\delta) = a/b$, find $\cot^2(\delta) + \cos^2(\delta)$:

A. $(2b^2 - a^2)/(b^2 - a^2)$

B. $a^4/[b^2(b^2 - a^2)]$ +++

C. $b^4/(a - b^2)$

D. $(a^2 - b)/b$

45. Given that $\tan(q) = 7/2$, evaluate $\sin(2q)$:

A. $28/35$

B. $35/28$

C. $28/53$ +++

D. $53/28$

46. Given that $\tan(q) = 7/2$, evaluate $\cos(2q)$:

A. $-53/45$

B. $53/45$

C. $45/53$

D. $-45/53$ +++

47. Given that $\tan(q) = 7/2$, evaluate $\tan(2q)$:

A. $-45/28$

B. $54/29$

C. $29/54$

D. $-28/45$ +++

48. Given that $\tan(q) = 7/2$, evaluate $\sin(2q) - \cos(2q)$:

A. $-53/73$

B. $54/29$

C. $73/53$ +++

D. $-28/45$

49. If $\sin(\gamma) = a/(1-b)$, find $\cot(\gamma)$:

A. $a/[(1-a-b)(1+a-b)]$

B. $(1-a-b)(1+a-b)/a$ +++

C. $b/[(1-a+b)(1+a-b)]$

D. $(1-a+b)(1+a-b)/b$

50. If $\sin(\gamma) = a/(1-b)$, find $\cos(\gamma)$:

A. $(1-a-b)(1+a-b)/(1-b)$ +++

B. $(1+a-b)(1+a-b)/(1-b)$

C. $(1-a-b)(1+a-b)/(1-b)$

D. $(1-a+b)(1+a-b)/(1-b)$

MTS 101 PLUS – Combined Past Questions (2)

SECTION A: SET THEORY, REAL NUMBER AND MATHEMATICAL INDUCTION

1. _____ is used to prove the validity of proposition of the set of non-negative integers.
 - A. Real number system
 - B. Real line
 - C. Associativity
 - D. The principle of mathematical induction +++
2. Which of the following statements is false for all positive integers using the principle of mathematical induction:
 - A. $2^n > n$
 - B. $5^{(2n)} + 3n + 1$ is a multiple of 9
 - C. $5^n - 1$ is a multiple of 4
 - D. $7^{(2n+1)} + 1$ is a multiple of 3 +++
3. A student was asked to show that $5^n - 1$ is divisible by 4 for all positive integers k using mathematical induction. Which statement is correct for all positive integers M?
 - A. $f(1) = 8$
 - B. $f(M) = 4(5^M + 1)$
 - C. $f(k+1) = 4(5^M + 1) +++$
 - D. $f(k) = 4(5^M + 1)$
4. Which of the following is not one of the properties of real numbers?
 - A. existence of the multiplicative inverse
 - B. closure
 - C. it satisfies the principle of mathematical induction +++
 - D. existence of the zero of the set
5. Let a, b, c and d be real numbers. Which of the following statements is false?
 - A. if $a > b$, then $a + c < b + c$ +++
 - B. if $a > b$ and $c > 0$, then $ac > bc$
 - C. if $a > b$ and $c < 0$, then $ac < bc$
 - D. if $a > b$ and $c > d$, then $a + c > b + d$

6. For what values of x can the formula $\sum_{i=0}^n x^i = \frac{1-x^{n+1}}{1-x}$ be proved?
- for all real numbers x
 - for all real numbers x satisfying $x \neq 1$ +++
 - for all real numbers x satisfying $x = 1$
 - for all positive integers n
7. $f(n) = n^4 + 4n^2 + 11$ is divisible by 16 for all odd positive integers. What are the possible values of n ?
- 1, 2, 3, ..., k , $k+1$ where k is a positive integer
 - 1, 2, 3, ..., k , $k+1$, ..., infinity where k is a positive integer
 - 1, 3, ..., $2k-1$, $2k+1$, ... where k is a positive integer +++
 - none of the above
8. In proving that $f(n) = 7^{(2n+1)} + 1$ is a multiple of 8, which of the following statements is correct?
- $f(k+1) = 344$
 - $f(k+1) = 8(49M - 6)$ where M is a positive integer +++
 - $f(k+1) = 8M - 1$ where M is a positive integer
 - $f(k+1) = 8(49M + 6)$ where M is a positive integer
9. Which of the following laws of algebra is incorrect for all x, y, z in N ?
- Closure: (i) $x+y$ in N (ii) $x.y$ in N
 - Commutative: (i) $x+y = y+x$ (ii) $x.y = y.x$
 - Associative: (i) $x+(y+z) = (x+y)+z$ (ii) $x.(y+z) = (x.y).z$ +++
 - Distributive: (i) $x.(y+z) = x.y + x.z$ (ii) $(y+z).x = y.x + z.x$
10. Given that for all positive integer values of n , $5^{(2n)} + 3n - 1$ is an integer multiple of 9. If k is a positive integer, then:
- $f(k+1) = 9(25M + 8k + 3)$ where M is a positive integer
 - $f(k+1) = 9(25M - 8k + 3)$ where M is a positive integer +++
 - $f(k+1) = 5^{(2(k+1))} + 3k + 2$ where M is a positive integer
 - $f(k) = 5^{(2(k+1))} + 3k + 2$ where M is a positive integer

[Questions 11-50 on Set Theory follow the same format. See Combined Past Questions (2) for full listing.]

SET THEORY QUESTIONS (Selected):

15. Given the set $A = \{a, b, c, d\}$; which of the following is NOT an element of the power set of A ?
- $\{a, b\}$
 - $\{\emptyset\}$ +++
 - A
 - $\{a, b, c\}$

16. How many subsets will a set containing 5 elements have?

- A. 32
- B. 25
- C. 36
- D. 64 +++

17. If $\mu = \{e, f, g, h, p, q, r, s\}$, $M = \{e, h, q, s\}$ and $N = \{h, p, r\}$. Find $M^c \cup N^c$:

- A. $\{f, p, r\}$
- B. $\{e, f, q, s\}$
- C. $\{e, f, g, p, q, r, s\}$ +++
- D. $\{e, g, p, q, r, s\}$

20. In a class of 40 students, 24 play football, 18 play volleyball and 6 do not play any game. How many Students play both Football and Volleyball?

- A. 2
- B. 7
- C. 8 +++
- D. 9

22. If $S = \{x: x^2 = 9, x < 2\}$ then S is equal to:

- A. $\{-3\}$ +++
- B. $\{3\}$
- C. $\emptyset$
- D. $\{-3, 3\}$

35. In a village all the people speak Hausa or English or both. If 56% speak Hausa and 63% speak English, what percentage speak both languages?

- A. 19% +++
- B. 119%
- C. 62%
- D. 38%

37. In a youth club with 94 members, 60 like modern music and 50 like traditional music. The number of members who like both traditional and modern music is thrice those who do not like any type of music. How many members like only one type of music?

- A. 8
- B. 24
- C. 62 +++
- D. 86

40. How many subsets will a set containing 6 elements have?

- A. 25
- B. 32
- C. 36
- D. 64 +++

47. $C = \{1, 2, 3, 4, \dots\}$. What is the name of the set C?

- A. Finite set
- B. Closed set
- C. Universal set
- D. Infinite set +++

50. $A \subseteq B$ and $B \subseteq C$ implies $A \subseteq C$ is the _____:

- A. Complementary law
- B. Transitivity Law +++
- C. Inverse law
- D. Commutativity law

MTS 101 PLUS – Combined Past Questions (123)

SECTION A: SET THEORY, REAL NUMBER AND MATHEMATICAL INDUCTION (Combined Set 123)

This document contains a comprehensive combined set of MTS 101 past questions covering all sections:

Section A: Set Theory, Real Number and Mathematical Induction (50 questions)

Section B: Circular Measure and Trigonometric Functions (50 questions)

Section C: Sequences and Series (50 questions)

Section E: Binomial Theorem and Expansion (50 questions)

All questions are formatted with four options (A, B, C, D) with the correct answer marked +++.

This compilation covers questions from multiple years and serves as a comprehensive revision tool for MTS 101 students.

Key topics covered in this combined document:

- Principle of Mathematical Induction (proving divisibility statements)
- Properties of Real Numbers (closure, commutativity, associativity, distributivity)
- Set operations (union, intersection, complement, symmetric difference)
- Venn diagram problems (two and three set problems)
- Counting problems using inclusion-exclusion principle
- Trigonometric identities (Pythagorean, double angle, compound angle)
- The t-formulas for half angles
- Circular measure (radian measure, arc length, sector area)
- Arithmetic Progressions (AP), Geometric Progressions (GP)
- Sum to infinity of geometric series
- Recurring decimals as fractions
- Pascal's triangle and binomial coefficients
- Binomial theorem for positive and negative indices
- Validity conditions for binomial expansions
- Quadratic equations (factor theorem, remainder theorem)

For full question listing, refer to the individual section documents.

MTS 101 PLUS – Mathematical Induction and Set Theory

SECTION A: SET THEORY, REAL NUMBER AND MATHEMATICAL INDUCTION

1. _____ is used to prove the validity of proposition of the set of non-negative integers.
 - A. Real number system
 - B. Real line
 - C. Associativity
 - D. The principle of mathematical induction +++
2. Which of the following statements is false for all positive integers using the principle of mathematical induction?
 - A. $2^n > n$
 - B. $5^{(2n)} + 3n + 1$ is a multiple of 9
 - C. $5^n - 1$ is a multiple of 4
 - D. $7^{(2n+1)} + 1$ is a multiple of 3 +++
3. A student was asked to show that $5^n - 1$ is divisible by 4 for all positive integers k. Which statement is correct for all positive integers M?
 - A. $f(1) = 8$
 - B. $f(M) = 4(5^M + 1)$
 - C. $f(k+1) = 4(5^M + 1) +++$
 - D. $f(k) = 4(5^M + 1)$
4. Which of the following is NOT one of the properties of real numbers?
 - A. existence of the multiplicative inverse
 - B. closure
 - C. it satisfies the principle of mathematical induction +++
 - D. existence of the zero of the set
5. Let a, b, c and d be real numbers. Which of the following statements is false?
 - A. if $a > b$, then $a + c < b + c$ +++
 - B. if $a > b$ and $c > 0$, then $ac > bc$
 - C. if $a > b$ and $c < 0$, then $ac < bc$
 - D. if $a > b$ and $c > d$, then $a + c > b + d$

6. For what values of x can the geometric sum formula be proved?
- for all real numbers x
 - for all real numbers x satisfying $x \neq 1$ +++
 - for all real numbers x satisfying $x = 1$
 - for all positive integers n
7. $f(n) = n^4 + 4n^2 + 11$ is divisible by 16 for all odd positive integers n . What are the possible values of n ?
- 1, 2, 3, ..., k , $k+1$
 - 1, 2, 3, ..., infinity
 - 1, 3, ..., $2k-1$, $2k+1$, ... +++
 - none of the above
8. In proving that $f(n) = 7^{(2n+1)} + 1$ is a multiple of 8:
- $f(k+1) = 344$
 - $f(k+1) = 8(49M - 6)$ +++
 - $f(k+1) = 8M - 1$
 - $f(k+1) = 8(49M + 6)$
9. Which law of algebra is incorrect for all x, y, z in N ?
- Closure
 - Commutative
 - Associative: $x.(y+z) = (x.y).z$ +++
 - Distributive
10. $f(k+1)$ for $5^{(2n)} + 3n - 1$ being a multiple of 9:
- $f(k+1) = 9(25M + 8k + 3)$
 - $f(k+1) = 9(25M - 8k + 3)$ +++
 - $f(k+1) = 5^{(2(k+1))} + 3k + 2$
 - $f(k) = 5^{(2(k+1))} + 3k + 2$
11. Sequential arrangement of mathematical induction:
- ii, i and iii
 - i, ii and iii
 - i, iii and ii +++
 - iii, ii and i
12. Given $1/(1.2) + 1/(2.3) + 1/(3.4) + \dots + 1/(n(n+1)) = n/(n+1)$. Then $f(k+1) = ?$
- $(k+3)/[(k+1)(k+2)]$ +++

B. $(k+3)/[(k+1)(k+3)]$

C. $1/[(k+1)(k+2)]$

D. $k/(k+1)$

13. Given $1/(1.3) + 1/(3.5) + 1/(5.7) + \dots + 1/((2n-1)(2n+1)) = n/(2n+1)$. Then $f(k+1) = ?$

A. $(k+1)/(2k+3) +++$

B. $k/(2k+3)$

C. $k/(2k+1)$

D. $1/[(2k+1)(2k+3)]$

14. Given sum formula involving powers of x . Then $f(k+1) = ?$

A. $1/(x-1) - 1/(x^{k+1}(x-1))$

B. $1/(x-1) - 1/(x^k(x-1))$

C. $(x^{k-1})/(x^{k+1}(x-1))$

D. $(x^{k+1}-1)/(x^{k+1}(x-1)) +++$

15. Given the set $A = \{a,b,c,d\}$; which of the following is NOT an element of the power set of A ?

A. $\{a,b\}$

B. $\{\emptyset\} +++$

C. A

D. $\{a,b,c\}$

16. How many subsets will a set containing 5 elements have?

A. 32

B. 25

C. 36

D. 64 +++

17. Find $M^c \cup N^c$ where $\mu = \{e,f,g,h,p,q,r,s\}$, $M = \{e,h,q,s\}$ and $N = \{h,p,r\}$:

A. $\{f,p,r\}$

B. $\{e,f,q,s\}$

C. $\{e,f,g,p,q,r,s\} +++$

D. $\{e,g,p,q,r,s\}$

18. P and Q are subsets of $\mu = \{x: x \text{ is an integer and } 1 < x < 15\}$. $P = \{x: x \text{ is odd}\}$ and $Q = \{x: x \text{ is prime}\}$.

Find the cardinal number of $(P^c \cap Q^c)$:

A. 3

B. 4

C. 5

D. 6 +++

19. If $P = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $Q = \{1, 4, 9\}$, $R = \{2, 4, 8\}$. Find $(P \cap Q) \cup R$:

A. $\{1, 2, 4, 8\}$

B. $\{1, 2, 4, 8, 9\}$ +++

C. $\{1, 2, 4, 7, 8\}$

D. $\{1, 2, 3, 4, 5, 6, 7, 8\}$

20. In a class of 40 students, 24 play football, 18 play volleyball and 6 do not play any game. How many Students play both Football and Volleyball?

A. 2

B. 7

C. 8 +++

D. 9

21. Four members of a school first eleven cricket team are also members of the first fourteen rugby team. How many boys play in only one of the two teams?

A. 25

B. 21

C. 17 +++

D. 29

22. If $S = \{x: x^2 = 9, x < 2\}$ then S is equal to:

A. $\{-3\}$ +++

B. $\{3\}$

C. $\emptyset$

D. $\{-3, 3\}$

23. A School of 38 teachers has 15 married women. If 6 of the teachers are couples, how many of the teachers are neither married nor have their spouses not in the school?

A. 17

B. 23

C. 20 +++

D. 21

24. Which of the following is equivalent to $[P^c \cup (Q \cap Q^c)]$?

A. P

B. P^c +++

C. Q

D. Q^c

25. Given that the sets A and B are partitioned sets of E; then $A \cap (E \cap B)^c$ is:

A. A +++

B. $\emptyset$

C. B

D. E

26. Which of the following sets is equivalent to $(P \cup Q) \cap (P \cap Q^c)^c$?

A. $P \cup Q$

B. $P \cup Q^c$

C. $P \cap Q^c$ +++

D. $\emptyset$

27. P and Q are subsets of $U = \{x: x^3 - x^2 - x + 1 = 0\}$. $P = \{x: x \text{ is an integer}\}$ and $Q = \{x: x \text{ is odd}\}$. Find $P \cap Q$:

A. $\{-1, 1, 1\}$

B. $\{-1, 1\}$ +++

C. $\{1, 1, -1\}$

D. $\{-1, -1\}$

28. If $E = \{1, 2, 3, 4\}$ and $A = \{1, 3, 5\}$, the symmetric difference $A \Delta E$ is:

A. $\{1, 2\}$

B. $\{2, 4, 5\}$ +++

C. $\{1, 2, 3, 4, 5\}$

D. $\{1, 3\}$

29. If $U = \{x: x \text{ is natural number and } 1 \leq x \leq 9\}$, $P = \{x: 1 \leq x < 4\}$ and $Q = \{2, 4, 6, 8\}$. Find $(P \cup Q)^c$:

A. $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

B. $\{1, 2, 3, 4, 6, 8\}$

C. $\{1, 5, 6, 7\}$

D. $\{5, 7, 9\}$ +++

30. In a science class of 41 students, each student offers at least one of Mathematics and Physics. If 22 Students offer Physics and 28 Students offer Mathematics, how many Students offer Physics only?

A. 19

B. 9

C. 13 +++

D. 14

31. If $U = \{0, 2, 3, 6, 7, 8, 9, 10\}$, $E = \{0, 6, 8, 10\}$ and $F = \{x: x^4=14\}$. Find $(E \cup F)^c$:

- A. $\{3, 7, 9\}$ +++
- B. $\{2, 3, 7, 9\}$
- C. $[3, 7, 9]$
- D. $\emptyset$

32. If $X = \{\text{all prime factors of } 44\}$ and $Y = \{\text{all prime factors of } 60\}$. The elements of $X \cup Y$ and $X \cap Y$ respectively are:

- A. $\{2, 4, 3, 5, 11\}$ and $\{4\}$
- B. $\{4, 3, 5, 1\}$ and $\{3, 4\}$
- C. $\{2, 5, 11\}$ and $\{2\}$
- D. $\{2, 3, 5, 11\}$ and $\{2\}$ +++

33. Simplify $(A \cup B)^c \cap (A \cap B^c)$:

- A. $(A^c \cup B)$
- B. $A \cup B^c$
- C. $(A \cup B)^c$
- D. $\emptyset$ +++

34. Simplify $(A \cup B)^c \cap A^c$:

- A. $(A \cup B)$
- B. $A \cap B^c$
- C. $A^c \cap B$
- D. $(A^c \cap B^c)$ +++

35. In a village all the people speak Hausa or English or both. If 56% speak Hausa and 63% speak English, what percentage speak both languages?

- A. 19% +++
- B. 119%
- C. 62%
- D. 38%

36. A set that contains another set is called:

- A. Power set
- B. Super set +++
- C. Subset
- D. Proper Set

37. In a youth club with 94 members, 60 like modern music and 50 like traditional music. The number of members who like both is thrice those who do not like any type. How many members like only one type?

- A. 8
- B. 24
- C. 62
- D. 86

38. The symmetric difference of A and B expressed as $A \oplus B$ is equal to:

- A. $(A-B) \cap (B-A)$
- B. $(A-B) \cup (B-A)$
- C. $(A \cap B) \cup (B \cap A)$
- D. $(A \cup B) \cap (B \cup A)$

39. $\xi = \{e, f, g, h, p, q, r, s\}$, $M = \{e, h, q, s\}$ and $N = \{h, p, r\}$. Find $M^c \cup N^c$:

- A. $\{f, p, r\}$
- B. $\{e, f, q, s\}$
- C. $\{e, f, g, p, q, r, s\}$
- D. $\{e, g, p, q, r, s\}$

40. How many subsets will a set containing 6 elements have?

- A. 25
- B. 32
- C. 36
- D. 64

41. Which of the following is equivalent to $\{P^c \cap (Q \cup Q^c)\}$?

- A. P
- B. P^c
- C. Q
- D. $\emptyset$

42. Which of the following options below is true?

- A. $A \cup A^c = A$
- B. $A \cap A^c = A$
- C. $(A^c)^c = U$
- D. $n(P(A)) = 2^{n(A)}$

43. Out of 25 teachers, 16 are married and 15 are women. If 6 of the men are married, how many of the women are not married?

A. 6

B. 10

C. 5 +++

D. 9

44. Which of the following is NOT a true property of a Cartesian product?

A. $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

B. $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$

C. $(A \cup B) \times C = (A \times C) \cup (B \times C)$

D. $A \times \emptyset = \emptyset \times A = A$ +++

45. If $(A \cap B^c) \cup (A^c \cap B) = A \Delta B$, what does the symbol " Δ " signify?

A. "difference"

B. "asymmetric difference"

C. "symmetric difference" +++

D. "union"

46. Let $U = \{1,2,3,4,5,6\}$, $A = \{1,2,3\}$ and $B = \{4,5,6\}$. Find $A^c \cup (B \cup B^c)$:

A. $\emptyset$

B. $\{1,2,3\}$

C. $\{4,5,6\}$

D. U +++

47. $C = \{1, 2, 3, 4, \dots\}$. What is the name of the set C?

A. Finite set

B. Closed set

C. Universal set

D. Infinite set +++

48. What does the set $\{x: x \notin A \text{ and } x \in B\}$ define?

A. set containing elements in A and not in B

B. set containing elements not in A but in B +++

C. set containing elements both in A and B

D. set containing elements both in A and B

49. When a set $C = \{\}$ it means that C is _____:

A. a universal set

B. C is finite set

C. C is an empty set +++

D. is a universal set

50. $A \subseteq B$ and $B \subseteq C$ implies $A \subseteq C$ is the _____:

A. Complementary law

B. Transitivity Law +++

C. Inverse law

D. Commutativity law

RESOLUTE FEM!

MTS 101 – Sequence and Series (CamScanner)

SECTION C: SEQUENCE AND SERIES

ARITHMETIC PROGRESSIONS

Definition: A sequence $a_1, a_2, a_3, \dots$ is an arithmetic progression (AP) if consecutive terms have a constant difference d (common difference).

General term: $U_n = a + (n-1)d$

Sum of first n terms: $S_n = n/2 [2a + (n-1)d] = n/2 [a + l]$

where a = first term, d = common difference, l = last term = $a + (n-1)d$

Number of terms: $n = (l - a)/d + 1$

GEOMETRIC PROGRESSIONS

Definition: A sequence $a_1, a_2, a_3, \dots$ is a geometric progression (GP) if consecutive terms have a constant ratio r (common ratio).

General term: $U_n = ar^{(n-1)}$

Sum of first n terms: $S_n = a(r^n - 1)/(r - 1)$ for $r > 1$

$S_n = a(1 - r^n)/(1 - r)$ for $r < 1$

Sum to infinity: $S_\infty = a/(1-r)$ for $|r| < 1$

HARMONIC PROGRESSIONS

If a, b, c are in HP, then $1/a, 1/b, 1/c$ are in AP.

The harmonic mean H of a and c is: $H = 2ac/(a+c)$

TRIANGULAR NUMBERS

The sums $1, 1+2, 1+2+3, \dots$ are called triangular numbers.

Sum of first n natural numbers = $n(n+1)/2$

SELECTED PAST QUESTIONS:

1. The first term of an arithmetic series is a , the common difference is d and the sum of all terms is S . Find the number of terms and the last term.

Answer: 20 terms, last term = 79

2. Find the sum to which the series $1/5 + 1/25 + 1/125 + \dots$ converges:

Answer: $\frac{1}{4}$ +++

3. What is the ninth term of the geometric sequence if the third term and the seventh term are $\frac{1}{9}$ and 81?

Answer: $\sqrt[9]{729}$ +++

4. What is the second term of the series, if the three numbers in geometric sequence whose sum is 13 and whose product is 64?

Answer: 4 +++

5. Evaluate the tenth term of the series 3, 9, 27, ...:

Answer: 59049 +++

6. Find the three numbers in arithmetic progression whose sum is 3 and whose product is -15:

Answer: -3, 1, 5 +++

7. Find the sum of all the odd numbers between 1 and 99:

Answer: 2500 +++

8. A model railway manufacturer makes pieces of track of lengths 8cm, 10cm, 12cm etc up to and including 38cm. An enthusiast buys 5 pieces of each length. What is the total length of track?

Answer: 345 +++

9. The first term of a geometric progression is 10 and the sum to infinity is 20. Find the common ratio:

Answer: $\frac{1}{2}$ +++

10. The sixth term of an arithmetic progression is 11 and the first term is 1. Find the common difference:

Answer: 2 +++

11. The fourth term of an AP is 13 while the tenth term is 31. Find the twenty-first term:

Answer: 64 +++

12. The second term and the fourth term of a GP are 8 and 32 respectively. What is the sum of the first four terms?

Answer: 60 +++

13. Express the recurring decimal 0.331 as a fraction in its lowest term:

Answer: $\frac{331}{999}$ +++

14. Find the sum of the first twenty-five odd numbers:

Answer: 625 +++

15. Insert three arithmetic means between 3 and 19. What is the sum of the linear function?

Answer: 55 +++

16. The sixth and thirteenth terms of an AP are 0 and 14 respectively. Find the sum of the first twenty terms:

Answer: -20 +++

17. To what sum does the series $1 - \frac{1}{5} + \frac{1}{25} - \dots$ converge?

Answer: $\frac{5}{6}$ +++

18. Find the sum to which the series $\frac{1}{6} + \frac{1}{12} + \frac{1}{24} + \dots$ converges:

Answer: 2 +++

19. Express the recurring decimal $0.3131\dots$ as a fraction in its lowest term:

Answer: $\frac{31}{99}$ +++

20. Express the recurring decimal $0.2727\dots$ as a fraction in its lowest term:

Answer: $\frac{27}{99}$ +++

RESOLUTE FEM!

MTS 101 – Quadratic Functions, Polynomials and Partial Fractions (CamScanner)

SECTION D & PARTIAL FRACTIONS: QUADRATIC FUNCTIONS, POLYNOMIALS AND PARTIAL FRACTIONS

QUADRATIC FUNCTIONS A quadratic function has the form $f(x) = ax^2 + bx + c$ where $a \neq 0$.

The graph of $f(x) = ax^2 + bx + c$ is a parabola:

- Opens upward if $a > 0$ (minimum point)
- Opens downward if $a < 0$ (maximum point)
- Vertex is at $x = -b/(2a)$

Completing the Square:

$$ax^2 + bx + c = a(x + b/(2a))^2 + (c - b^2/(4a))$$

$$\text{Minimum/Maximum value} = c - b^2/(4a) \text{ at } x = -b/(2a)$$

QUADRATIC FORMULA

For $ax^2 + bx + c = 0$:

$$x = (-b \pm \sqrt{b^2 - 4ac}) / (2a)$$

Discriminant $D = b^2 - 4ac$:

$D > 0$: two distinct real roots

$D = 0$: two equal real roots

$D < 0$: no real roots

VIETA'S FORMULAS (Roots of $ax^2 + bx + c = 0$)

Sum of roots: $\alpha + \beta = -b/a$

Product of roots: $\alpha * \beta = c/a$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2(\alpha * \beta)$$

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3(\alpha * \beta)(\alpha + \beta)$$

POLYNOMIALS A polynomial of degree n : $P(x) = a_n x^n + a_{(n-1)} x^{(n-1)} + \dots + a_1 x + a_0$

Factor Theorem: $(x - a)$ is a factor of $P(x)$ if and only if $P(a) = 0$.

Remainder Theorem: When $P(x)$ is divided by $(x - a)$, the remainder is $P(a)$.

Polynomial Long Division:

To divide $P(x)$ by $D(x)$: $P(x) = Q(x) \cdot D(x) + R(x)$

where $Q(x)$ is the quotient and $R(x)$ is the remainder, with degree of $R < \text{degree of } D$.

PARTIAL FRACTIONS

Proper fraction: degree of numerator $<$ degree of denominator.

Improper fraction: degree of numerator $\geq$ degree of denominator.

Case 1 - Distinct linear factors in denominator:

$$f(x)/[(x-a)(x-b)] = A/(x-a) + B/(x-b)$$

Case 2 - Repeated linear factor:

$$f(x)/[(x-a)^2(x-b)] = A/(x-a) + B/(x-a)^2 + C/(x-b)$$

Case 3 - Irreducible quadratic factor:

$$f(x)/[(x^2+px+q)(x-a)] = (Ax+B)/(x^2+px+q) + C/(x-a)$$

Case 4 - Improper fractions: First perform polynomial division, then decompose the proper fraction part.

SELECTED PAST QUESTIONS:

1. If α and β are roots, sum = 5. Find $\alpha + \beta$:

Answer: 8 +++

2. Find the quotient when $f(x)$ is divided by $g(x)$:

(See detailed solutions in solution booklet)

3. Find in terms of k the remainder when $f(x)$ is divisible by $(x - k)$:

Answer: Use remainder theorem, substitute $x = k$ +++

4. The solution to the quadratic equation:

Answer: Use quadratic formula with appropriate a, b, c values +++

5. Solve -6, 3 for the quadratic:

A. -6, 3 +++

6. Find the value of n when $C(n,2) = 120$:

A. 16 +++

7. The minimum value of y in the equation $ax^2 + bx + c$:

Answer: -1 +++

8. Factorize $x^3 - 6x^2 + 11x - 6$:

Answer: $(x-1)(x-2)(x-3)$ +++

9. Solve $x^2 - x - 6 = 0$:

Answer: $x = 3$ and $x = -2$ +++

10. If $(x-2)$ and $(x+3)$ are factors, find the third factor using long division +++

11. Find partial fractions of $(3x+5)/[(x-3)(2x+1)]$:

Answer: $2/(x-3) - 1/(2x+1)$ +++

12. Express $(3x+1)/[(x-1)^2(x+2)]$ in partial fractions:

Answer: $5/[9(x-1)] + 4/[3(x-1)^2] - 5/[9(x+2)]$ +++

13. For improper fractions, first perform polynomial division before partial fractions +++

PRACTICE PROBLEMS (from CamScanner document)

These pages contain hand-written worked examples and practice problems on:

- Graphing quadratic functions
- Finding axis of symmetry and vertex
- Solving polynomial equations using factor theorem
- Polynomial long division
- Setting up and solving partial fraction decompositions

Students are advised to practice these problems independently before consulting solutions.