



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2025_1 EXAMINATION...

Course Code: MTH301

Course Title: Functional Analysis I

Credit Unit: 3

Time Allowed: 3 Hours

Total: 70 Marks

Instruction: Answer Question One (1) and Any Other Three (3) Questions

1. (a) Define a topology τ on X . **(4 marks)**
(b) Give an example in each of discrete and indiscrete topology. **(5 marks)**
(c) Let X be a complete metric space and $\{O_n\}$ is countable collection of dense open subsets of X . Show that $\bigcup O_n$ is not empty. **(11 marks)**
(d) Let $K \subseteq X$ be compact. Show that K is bounded. **(5 marks)**

2. (a) The collection Z_d defined as $Z_d = \{A \subseteq X: x \in A \text{ implies there exists } r > 0 \text{ such that } B(x, r) \subseteq A\}$ is a topology on X , known as the topology induced by the given metric d . In a metric space (X, d) for each $x \in X, r > 0$, show that $B(x, r)$ is an open subset of (X, Z_d) . **(7 marks)**

(b) Let K be a collection of nonempty closed subsets of a compact space T such that every finite sub collection of K has a nonempty intersection. Show that the intersection of all sets from K is non-empty. **(8 marks)**

3. (a) State Heine-Borel theorem. **(3 marks)**
(b) Show that a continuous image of a compact space is compact. **(12 marks)**

4. (a) State axioms of addition of a real number system $(\mathbb{R}, +, \cdot)$ **(5 marks)**
(b) Prove that a subspace T of a topological space S is disconnected iff it is separated by some open subsets U, V of S . **(10 marks)**

5. Let (X, d) and (Y, d_1) be metric spaces and g is a mapping of X into Y . Let τ and τ_1 be the topologies determined by d and d_1 respectively. Show that $g(x, \tau) \rightarrow (y, \tau)$ is continuous if and only if $x_n \rightarrow x \Rightarrow g(x_n) \rightarrow g(x)$: that is if $x_1, x_2, \dots, x_n, \dots$ is a sequence of points in (X, d) converging to x , then the sequence of points $g(x_1), g(x_2), \dots, g(x_n), \dots$ in (Y, d_1) converges to $g(x)$. **(15 marks)**

6. Prove that a set A is a closed set if and only if it contains all its limit points. **(15 marks)**