NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, NnamdiAzikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
January\February Examination 2018

Course Code: MTH302
Course Title: Elementary Differential Equations II
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Iinstruction: Answer Question one and Any other4 Questions

1. (a) Show that the solution of $\frac{dy}{dx} = y$ around $x = 0$ is $y = Ae^x$, using the power series solution **(6 marks)**

   (b) Consider the series solution of $P(x)\frac{d^2y}{dx^2} + Q(x)\frac{dy}{dx} + R(x)y = 0$ around$x = a$. State the condition, in which$x = a$ can be classified as an ordinary point, a regular singular and an irregular singular point.**(6 marks)**

   (c) Find the series solution of$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, which is an ordinary point$x = 0$. **(10 marks)**

2. Use the Frobenius method of the form $y(x) = \sum_{n=0}^{\infty} a_n\,(x-a)^{n+r}$to solve $4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$, which is a regular singular point at $x = 0$.**(12 marks)**

3. Find the series solution of $\frac{d^3y}{dx^3} - xy = 0$, which is an irregular singular point at $x = 0$ **(12 marks)**

4. (a) Solve the initial value problem $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0,\ y(0) = 1, y'(0) = 2$**(8 marks)**

   (b) Use the complementary solution of 4 (a), obtain the general solution $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x^2 + x + 1$**(4 marks)**

5. (a) The beta function $B(m,n)$is defined by$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$, then provethat $B(m,n) = 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta\,\cos^{2m-1}\theta\,d\theta$ **(8 marks)**

   (b)Use the definition beta function in 5(a), obtain solution of $\int_0^1 x^7(1-x)^5\,dx$**(4 marks)**

6 (a) Obtain the Fourier Cosine series for $f(x) = \frac{1}{2}x^2\ ,\ -\pi < x < \pi$.

Hence deduce that $\sum_{n\to 0}^{\infty}\frac{(-1)^n}{n^2} = \frac{\pi^2}{12}$ **(9marks)**

(b) Suppose that $f(x)$ is odd obtain the general Fourier Sine Series. **(3 marks)**