NATIONAL OPEN UNIVERSITY OF NIGERIA

University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES

DEPARTMENT OF MATHEMATICS

2025_2 EXAMINATIONS

Course Code: MTH312
Course Title: Abstract Algebra
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other Three (3) Questions

1a(i). Let G be a group such that $N \leq G$, when is N said to be a normal subgroup of G? (2 marks).

(ii). If $N \lhd G$, define the left quotient set of G by N. (1.5marks).

(iii). If $N \lhd G$, define the right quotient set of G by N. (1.5marks).

(b). If H is a subgroup of a group, G, prove that the following statements are equivalent. G

(i). H is normalin G.

(ii). $g^{-1}Hg \subseteq H \forall g \in G.$ (9marks).

(iii). $g^{-1}Hg = H \forall g \in G.$

1(c). Show that every subgroup of Z is normal in Z (hint: Z is written additively).(7marks).

1d(i). Show that every subgroup of a commutative group is normal in G. (3marks).

(ii). The Lagrange's Theorem as given below is not correct. Write the Theorem correctly.

Lagrange's Theorem. The order of a subgroup, H of a group, G divides the order of the group.(1marks).

2(a). Let G be a group and H a subgroup of G such that $H \lhd G$, then define the following

(i) Left Quotient set of G by H. ($1\frac{1}{2}$ marks).

(ii) Right Quotient set of G by H. ($1\frac{1}{2}$ marks).

(iii) Given that G and H are groups such that $H \lhd G$, define what is meant by the Index of G in H. ($1\frac{1}{2}$ marks).

b. Show that the set of all left cosets of H in G is a group under cosets multiplication. (8marks).

c. Explain that if $H \lhd G$, then $G/H = G \backslash H$. (2.5 marks).

3(a). Let $(G_1,*)$ and $(G_2,\circ)$ be two groups. Then define a homomorphism between the two groups. (2marks).

(b). If σ is a homomorphism from G_1 to G_2, then define (i). kerσ. (2marks).
(ii). Imσ. (2marks).

c. Consider the two groups, and ($(\mathrm{R},+)$) and ($(R \backslash \{0\})$, (i).Show that the map, $\exp:$($(\mathrm{R},+)$) $\rightarrow$($(R \backslash \{0\})$: $\exp(r) = e^r$ is a group homomorphism. (6marks)

(ii). Find Ker exp. (1.5marks). (iii). FindIm exp. (1.5marks).

4a(i). what is a ring? (5marks, 1 for each property.)

What do we mean by $M_{22}(R)$? (1.5 marks). Hence show that

($M_{22}(Z)$,+,.) is a ring. (6.5 marks).

4b(i). In terms of algebraic operations, differentiate between a group and ring. (1mark)

(ii).Write 'yes' or 'no'. 'It is the multiplicative property that is used to describe a ring'. (1mark).

5a(i). What is a subring? (3mks). (ii). List the elements of $3Z_6$ (2mks).Show that $3Z$ is a subring. (6mks).

b. Let R be a ring and $A \subseteq R$, When is A called an ideal of R. (6mks).

6. Q6a(i). What do we mean by the alternating group in S_n? (2marks).

(ii). What is the notation for the alternating group in S_n? (1mark).

(b).Write down all the elements of A_n. (3marks)
(c). State without prove, the Cayley's Theorem. (2marks).
(d). Check if $(S_3,\circ)$ is commutative. (7marks).