



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2025_2 EXAMINATIONS

Course Code: MTH381

Course Title: Mathematical Methods III

Credit Unit: 3

Time Allowed: 3 Hours

Total: 70 Marks

Instruction: Answer Question One (1) and Any Other Three (3) Questions

QUESTION ONE

(a) (i) If $u = xyz$, $v = x^2 + y^2 + z^2$, $w = x + y + z$, find $J = \frac{\partial(u,v,w)}{\partial(x,y,z)}$

(ii) Given the functions: $u = x^2 + y^2$, $v = 2xy$, determine the Jacobian. **8marks**

(b) Define the Jacobian of a functions u and v hence find the Jacobian of $u = x + \frac{y^2}{x}$ and

$$v = \frac{y^2}{x}. \quad \textbf{5marks}$$

(c) Find the Laplace transform of $f''(t)$. **5marks**

(d) Let $H(s) = \frac{1}{(s^2 + w^2)^2}$ find $H(t)$. **7marks**

QUESTION TWO

(a) Define the dependence and independence of a function. **3marks**

(b) If $u(x) = \sin bx$ and $v(x) = \cos bx$ determine whether the functions $u(x)$ and $v(x)$ are linearly dependent or independent. **6marks**

(c) Evaluate the integral $\int_0^1 \int_0^1 (x^2 + y^2) dy dx$. **6marks**

QUESTION THREE

(a) Evaluate $\oint_c \vec{f} \cdot d\vec{r}$ by Stoke's theorem where $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ and c is the boundary of the rectangle $x = \pm a$, $y = 0$ and $y = b$. **5marks**

(b) Find the Fourier series expansion of periodic function of period 2π , defined by

$$f(x) = \begin{cases} x & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \pi - x & \text{if } \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases} \quad \textbf{6marks}$$

(c) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{\sqrt{x^2 + y^2}} dy dx$ by changing to polar coordinate. **4marks**

QUESTION FOUR

- (a) Find the volume cut off from the paraboloid $x^2 + \frac{y^2}{4} + z = 1$ by the plane $z = 0$. **6marks**
- (b) Using Stoke's theorem or otherwise, evaluate $\int_c [(2x - y)dx - yz^2 dy - y^2 z dz]$ where c is the circle $x^2 + y^2 = 1$ corresponding to the surface of sphere of what unit radius. **4marks**
- (c) Using the divergence theorem show that $\iint_s \nabla(x^2 + y^2 + z^2) \cdot \vec{ds} = 6v$ where s is closed surface enclosing volume V. **5marks**

QUESTION FIVE

- (a) State Green's and Stoke's theorems without proof. **4marks**
- (b) The vector field $\vec{F} = x^2 \hat{i} + z \hat{j} + yzx \hat{k}$ is defined over the volume of the cuboid given by $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$, enclosing the surface. Evaluate the integral surface $\iint_s \vec{F} \cdot d\vec{s}$. **7marks**
- (c) Evaluate $\iiint \frac{dx dy dz}{x^2 + y^2 + z^2}$ throughout the volume of the sphere $x^2 + y^2 + z^2 = a^2$ **4marks**