



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2025_2 EXAMINATIONS

Course Code: MTH401

Course Title: GENERAL TOPOLOGY I

Credit Unit: 3

Time Allowed: 3 Hours

Total: 70 Marks

Instruction: Answer Question One (1) and Any Other 3 Questions

- 1a. Define a topological space X and explain with an example. 7marks
- b. Let $\{F_i\}_{i \in I}$ be a collection of closed sets in a topological space (X, τ) . Prove that the intersection of any collection of closed sets is closed. 8marks
- c. Define homeomorphism. 2marks
- d. Given a bijective function $f: X \rightarrow Y$, between two topological spaces X and Y , prove that the homeomorphism between the spaces preserve compactness. 8marks
- 2a. Let $f: X \rightarrow Y$ be a continuous function. Prove that a continuous function f from a compact space to a Hausdorff space is closed and proper. 6marks
- b. Define and prove the Heine-Borel theorem. 9marks
- 3a. Let $f: X \rightarrow Y$ be a continuous function. Prove that the continuous image of the connected space is connected. 7marks
- b. Prove that every metric space is a Hausdorff space. 8marks
- 4a. Define compactness of a topological space, say X . 2marks
- b. Prove that a closed subset of a compact space is compact. 5marks
- c. Prove that the intersection of any collection of closed sets is closed. 8marks
- 5a. Define a topological property and give two examples of topological properties. 3marks
- b. Prove that a space is connected if and only if the only clopen sets are the empty set and the entire space. 6marks
- c. Define a basis of a topology and explain with an example. 6marks
- 6a. State the Pasting lemma on union of closed sets. 8marks
- b. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by 7marks
- $$f(x) = x^2 + 1, \text{ if } x \leq 0 \text{ and } f(x) = \frac{1}{2}(x + 2), \text{ if } x \geq 0.$$
- Show that f is continuous on $\mathbb{R}$.