



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2025_2 EXAMINATIONS

Course Code: MTH402

Course Title: Topology II

Credit Unit: 3

Time Allowed: 3 Hours

Total: 70 Marks

Instruction: Answer Question One (1) and Any Other Three (3) Questions

1a. Define a topology on X

b. Define convergent sequence.

c. (i) Define a dense Set and Everywhere Dense

(ii) Define a metrizable topological space

d. (i) Define Neighborhood of a point

(ii) Define Limit Point Compactness

e. (i) Define Lebesgue number of a Topological Space

2. Let (X, τ) and (Y, τ^*) be topological spaces.

a. When is a function

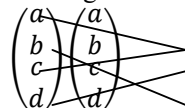
$f : (X, \tau) \rightarrow (Y, \tau^*)$ said to be continuous?

b. Prove the result below:

Let $f : (X, \tau) \rightarrow (Y, \tau^*)$ and $g : (Y, \tau^*) \rightarrow (Z, \tau^{**})$ be continuous. Then, the composition function $g \circ f : (X, \tau) \rightarrow (Z, \tau^{**})$ is also continuous.

c. Consider the following topology defined on $X = \{a, b, c, d\}$

$\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}\}$. Let the function $f : (X, \tau) \rightarrow (X, \tau)$ be defined by the diagram



(i) Show that f is not continuous at c

(ii) Show that f is continuous at d .

3. Prove the result below

a. A topological space (X, τ) is connected if and only if it can not be expressed as the union of two non-empty sets that are separated in X .

b. Consider $\mathbb{R}$ with the left limit topology. Is the interval $[0, 1]$ connected?

4. Let X, Y be topological spaces with X compact and Y Hausdorff.

a. Prove that a continuous map $f : X \rightarrow Y$ must be closed.

b. Prove that a continuous bijection $f : X \rightarrow Y$ must be a homeomorphism.

5. Prove the result below

a. The real line $\mathbb{R}$ endowed with the standard topology is not compact.

b. Consider $\mathbb{R}$ with the left limit topology. Is the interval $[0, 1]$ compact?

6(a). Consider the topology $\tau = \{X, \emptyset, \{a\}, \{a, c, d\}, \{c, d\}, \{b, c, d, e\}\}$ on $X = \{a, b, c, d, e\}$ and the subset $A = \{a, d, e\}$ of X . List the relative topology on A .

6.(b) Given that X and Y be two topological spaces. And let $f : X \rightarrow Y$ be a function. Suppose X is first countable. If for every sequence $\{x_n\}$ of X such that $x_n \rightarrow x$ in X as $n \rightarrow \infty$, one has that $f(x_n) \rightarrow f(x)$ in Y . Prove that f is continuous.