



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2025_2 EXAMINATIONS

Course Code: MTH411

Course Title: Measure Theory

Credit Unit: 3

Time Allowed: 3 Hours

Total: 70 Marks

Instruction: Answer Question One (1) and Any Other Three (3) Questions

Question 1

- a. Define the following Outer Measure and Inner Measure
- b. Explain the terms
 - i. f is L-measurable set and
 - ii. Length of a set A
- c. State Beppo Levi theorem.
- d. Suppose $f \in \mathcal{A}[a, b]$. Given any $\epsilon > 0$, there exists a $\delta > 0$. Prove that if $m(E) < \delta$, then $|\int_E f| < \epsilon$.

Question 2

- a. Define σ -algebra (sigma algebra). (2marks)
- b. Prove that if A_1 and A_2 are measurable sets then $A_1 \cup A_2$ and $A_1 \cap A_2$ are measurable sets. (13marks)

Question 3

- a. With a clearly stated definition distinguish between
 - i. Upper sum and upper integral (4marks)
 - ii. Lower sum and lower integral (4marks)
- b. Show that if a closed set F is contained in an open interval Δ , then $m(F) = m(\Delta) - m(\Delta - F)$. (7marks)

Question 4

- a. Explain the term simple function. (3marks)
- b. Let $\{f_n\}$ be a sequence of non-negative functions in $\mathcal{A}[a, b]$ suppose $\inf_{x \rightarrow \infty} f(x) = f(x)$ a.e in $[a, b]$ show that:
 - i. If $f \in \mathcal{A}[a, b]$, then $\int_a^b f \leq \inf_{x \rightarrow \infty} \int_a^b f_n$ (9marks)
 - ii. If $f \notin \mathcal{A}[a, b]$, then $\lim_{x \rightarrow \infty} \inf \int_a^b f_n = +\infty$ (3marks)

Question 5

- a. Explain bounded function and Lebesgue integrable bounded function. (5 marks)
- b. Show that the function $f(x) = \begin{cases} 1, & x \in [0,1], x \text{ is rational} \\ 0, & x \in [0,1], x \text{ is irrational} \end{cases}$ (10marks)
is Lebesgue integrable but not Riemann integrable.

Question 6

- a. State (i) Monotone Convergence theorem. (3 marks)
- (ii) Dominated Convergence theorem. (3 marks)
- b. Let $(X, \mathcal{M}, \mu)$ be a measure space, and let f and g be extended real-valued functions on X that are equal almost everywhere. If μ is complete and if f is measurable, explain that g is measurable. (9 marks)