



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2023_1 POP EXAMINATION

Course Code: MTH421
Course Title: Ordinary Differential equations
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question Number One and any Other Three Questions

1. (a) Solve the following initial value problems

(i) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0, \quad y(0) = 4, \quad y'(0) = 1$ (9 marks)

(ii) $x^2 \frac{d^2y}{dx^2} - 6y = 0, \quad y(1) = 2, \quad y'(1) = 1$ (9 marks)

(b) Verify that equation $3x(xy - 2)dx + (x^3 + 2y)dy = 0$ is exact and find the general solution (7 marks)

2. Solve the non-homogenous linear differential equation

(i) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x^2$ (9 marks)

(ii) $\frac{dy}{dx} - y^2 = 1$ (6 marks)

3. (a) Prove that $x = 3e^{7t}$, $y = 2e^{7t}$ and $x = e^{-t}$, $y = -2e^{-t}$ are solution of a system of differential equations (6 marks)

(b) Find the solution to the boundary value problem
 $y'' + 4y = 0, \quad y(0) = 1, \quad y\left(\frac{\pi}{2}\right) = 0$ (9 marks)

4. (a) Show that $y = e^{-2x}$ is a solution to the equation
 $y'' + y' - 2y = 0$ (6 marks)

(b) Solve the equation $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$ (9 marks)

5. (a) Verify that equation $(4xy + 3y^2 - x)dx + x(x + 2y)dy$ is /not exact, then solve the equation (10 marks)



(b) let A be the matrix given by: $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix}$.

Find

- (i) The eigenvalues **(2 ½ marks)**
- (ii) The generalized eigenspaces **(2 ½ marks)**

6. (a) Find the solution in explicit form for equation

$$\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)}, \quad y(0) = -1 \quad \textbf{(9 marks)}$$

(b) Find the eigenvalues and eigenfunctions of the Sturm-Liouville problem

$$\frac{dy^2}{dx^2} + \lambda y = 0, \quad 0 \leq x \leq L, \quad y(0) = 0, \quad y(L) = 0 \quad \textbf{(6 marks)}$$