NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2024_2 EXAMINATION

Course Code: MTH 422 **Course Title: Partial Differential Equations**
Credit Unit: 3 **Time Allowed: 3 Hours**
Total: 70 Marks
Instruction: Answer Question One and any Other Three Questions

1. Consider the following second-order linear partial differential equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{4\partial^2 u}{\partial y^2} - 2u = 0,$$

subject to the following boundary conditions:

i. $u(0, y) = 0$ for all $y \in [0, 1]$,

ii. $u(1, y) = 0$ for all $y \in [0, 1]$,

iii. $u(x, 0) = \sin(\pi x)$ for $x \in [0, 1]$

(a) Find the general solution of the PDE. **(15marks)**

(b) Use the given boundary conditions to determine the specific solution. **(10 marks)**

2. Solve the non-homogeneous Cauchy problem

$u_{tt} - u_{xx} = t^7; \quad u(x, 0) = 2x + \sin x\,; \; u_t(x, 0) = 0$ **(15marks)**

3(a) Using the following solution $u(x, t) = F(x + ct) + G(x - ct)$, solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ such that $u(x, 0) = e^{-x^2}, \frac{\partial u}{\partial t}(x, 0) = 0, -\infty < x < \infty.$ **(9 marks)**

(b) Solve $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$; $u(0, t) = 0\ for\ t \geq 0\ and\ u(L, t) = 0\ for\ t \geq 0$ and initial conditions, $u(x, 0) = f(x)\ for\ \ 0 \leq x \leq L$ and $\frac{\partial u}{\partial t}(x, 0) = g(x)\ for\ \ 0 \leq x \leq L$ such that

$$f(x) = \frac{1}{2}\sin\frac{\pi x}{L} + \frac{1}{4}\sin\frac{3\pi x}{L} + \frac{2}{5}\sin\frac{7\pi x}{L}, \quad g(x) = 0$$ **(6 marks)**

4. Find the general solution (in terms of arbitrary function) of the first order PDE

$2u_x(x, y) + 3u_y(x, y) + 8u(x, y) = 7y$ **(15marks)**

5. Consider the equation

$$u_{tt} - 9u_{xx} = 0, \ -\infty < x < \infty, \ t > 0$$

$$u(x,0) = f(x) = \begin{cases} 1, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$$

(15 marks)

$$u_t(x,0) = g(x) = \begin{cases} 1, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$$

Evaluate $u(0, \frac{1}{6})$ and find $\lim_{t \to \infty} u(x,t)$.